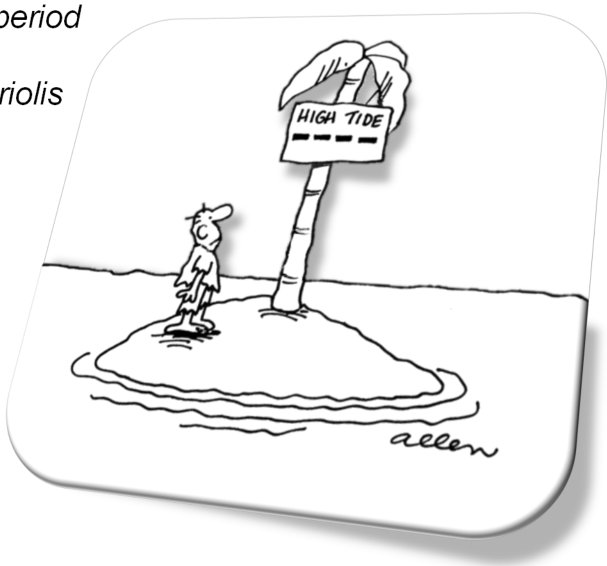
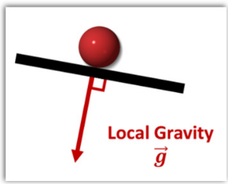
Chapter5: Tides

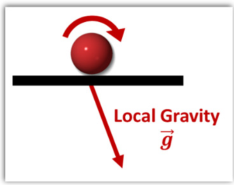
- The geoid and the mean Sea-Level
- Tidal frequencies
- Tidal forces and semi-diurnal period
- Lunar and Solar tides
- Amphidromic systems and Coriolis deflection

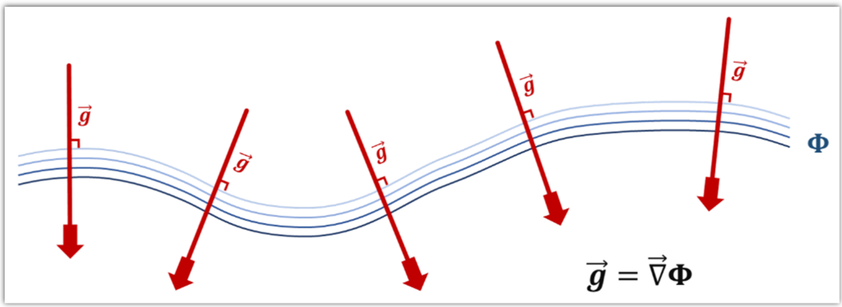


The equilibrium sea surface

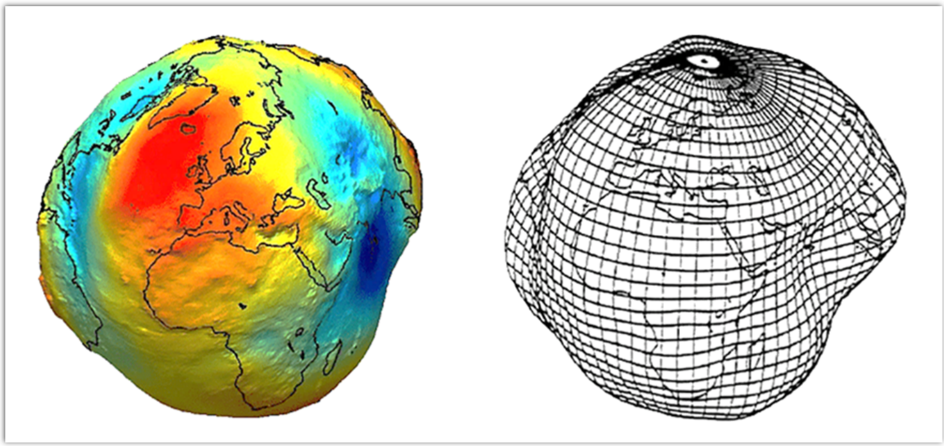
⇨ Imagine a surface to which the gravitational force is always perpendicular.
↳ A ball would sit on this surface without rolling.
⇨ It is a surface of equal gravitational potential. It is called a **geoid**.
Waves, tides and geostrophic steady currents are all perturbations to the geoid.





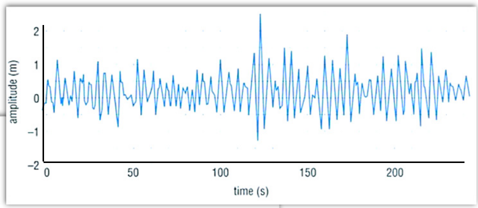


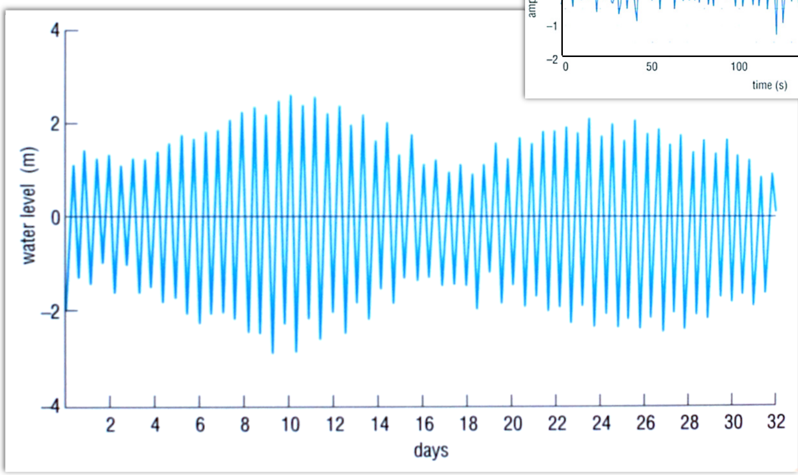
The geoid



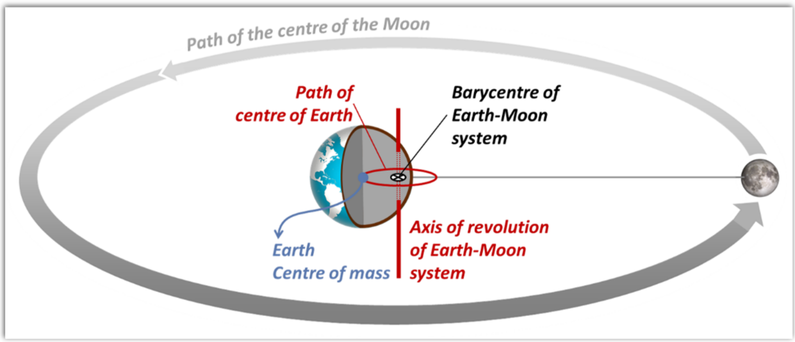
A typical tide gauge reading

It is immediately obvious that there are at least two frequencies involved here !
Here we see a semi-diurnal tide modulated semi-monthly.
What sets these frequencies (and amplitudes) ?
How do they vary from one location to another ?



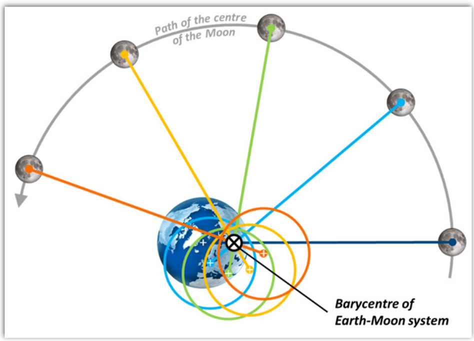


Moon's orbital motion



⇨ The earth and the moon rotate around their common centre of mass in $27.3\ days$ (a sideral month)

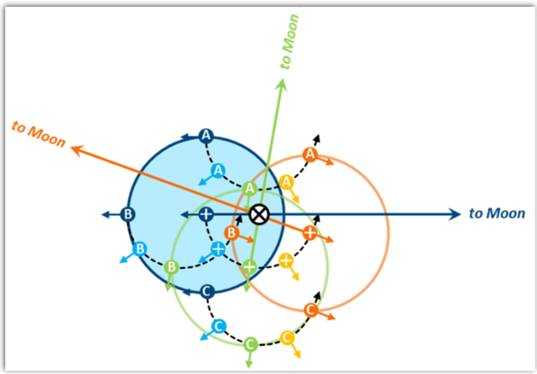
⇨ The barycenter of the Earth-Moon system lies within the earth, at $4700\ km$ from its centre ($a = 6400km$).

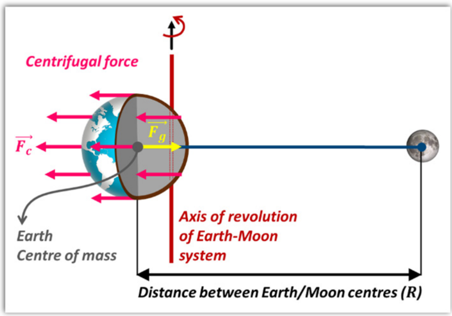


Forces at work: The centrifugal force

⇨ **<u>The centrifugal force</u>** acting on the Earth-Moon system balances the forces of gravitational attraction between the two bodies, so that the moon is not escaping from or falling into the earth.

⇨ The earth translates in a circuit around the barycentre of the Earth-Moon system, such that any point within or on the surface of the earth describes the same trajectory





⇨ The **magnitude of the uniform centrifugal force** equals the gravitational force between the earth and the moon centers of mass.

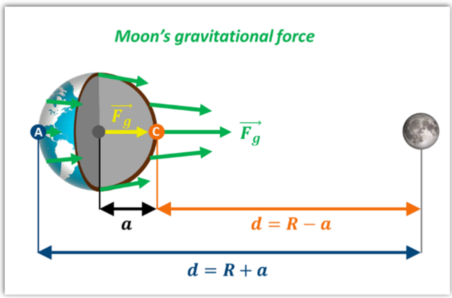
$$Fc = \frac{GM_1M_2}{R^2}$$

G the universal gravitational constant ($G = 6.672 \times 10^{-11}\ N\ m^2\ kg^{-2}$)

Forces at work: The gravitational force

⇨ **The magnitude of the gravitational force** produced by the moon upon the earth is not the same at different positions on the earth because the magnitude of the gravitational force exerted **varies with the distance** to the attracting body. **And *this is where the subtilty lies*.**

$$Fg = \frac{GM_1M_2}{d^2}$$



⇢ Points nearest the moon (C) experience a greater gravitational pull from the moon than the ones on the opposite side of Earth (A).

⇢ The direction of the moon's gravitational pull is directed towards the centre of mass of the moon. It is not exactly parallel to the direction of the centrifugal force, except along the line joining the centres of Earth and Moon.

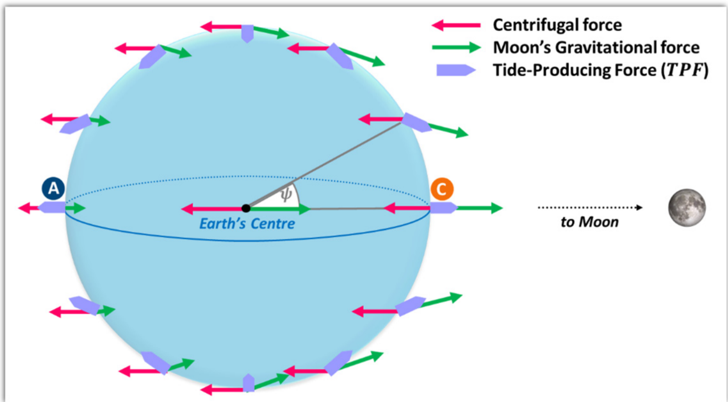
The Tide-Producing Force

⇨ The net force at point C is due to attraction of the moon minus the centrifugal force due to rotation about the common centre of mass.

$$\frac{GM_1M_2}{(R-a)^2} - \frac{GM_1M_2}{R^2}$$

The only place in balance is the centre of the earth

⇨ The TPF in C is equal to $\frac{GM_1M_2a(2R-a)}{R^2(R-a)^2} \approx \frac{GM_1M_22a}{R^3}$

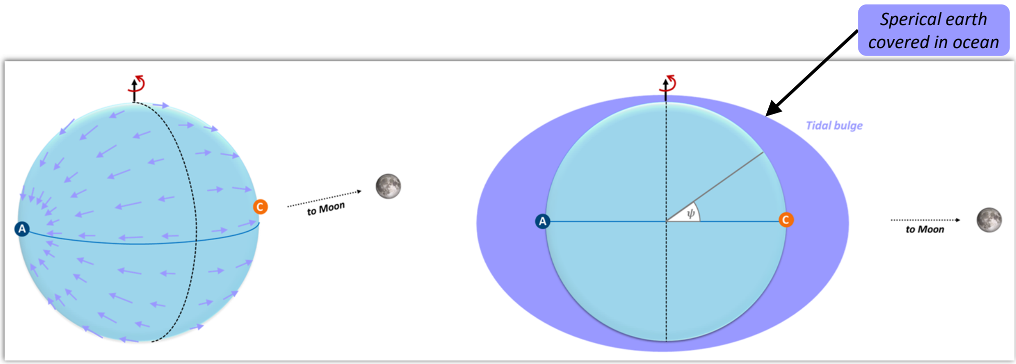


☝The forces that generate the tide (centrifugal and gravitational forces) follow $1/R^2$ laws, while the Tide-Producing Force is proportional to $1/R^3$.

The equilibrium tide

⇨ The pull of the Tide-Producing Force on the ocean is negligible compared to the earth's gravity, but where it pulls horizontally it can have an effect.

↳ This distorts the ocean into a bulge called the "equilibrium tide"



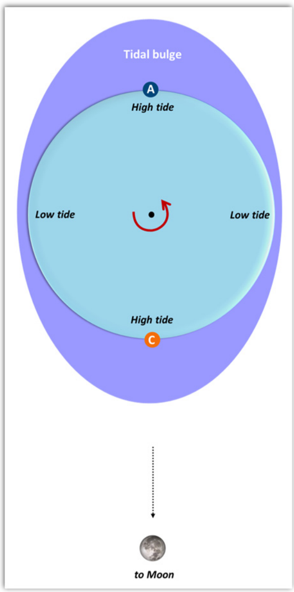
What is the wavelength of the tidal wave?

Tidal period

⇨ The Earth rotates within the **tidal bulge** producing two tides per day at the equator.

⇨ But of course it's not as simple as this, because the moon also moves.

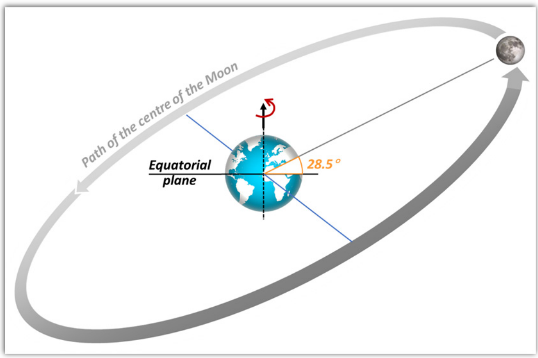
↳ So the equilibrium tidal period is actually 24h and 50 minutes

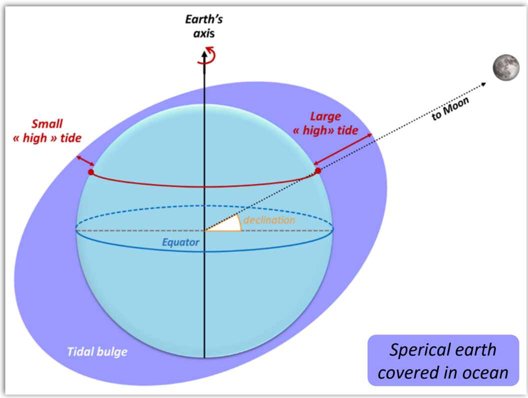


Is it actually possible for the tidal bulge to travel this fast at the equator ?

Tropic and Equatorial tides

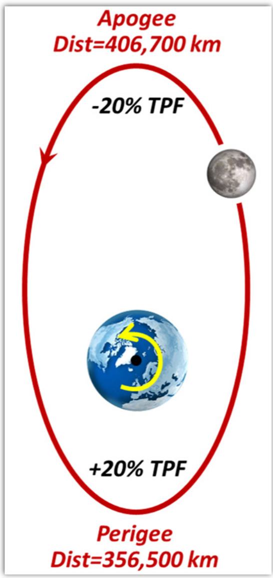
⇨ Things are further complicated by the fact that the **moon's orbit is tilted** with respect to the equator by 28.5°





⇨ This leads to an **asymmetry** between the two high tides of the day, which is itself a function of the time of the lunar month.

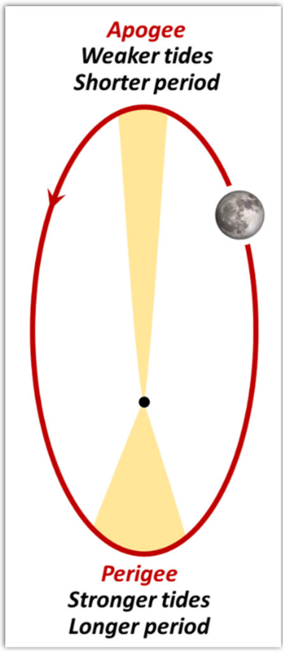
Moon's elliptical orbit



⇨ Things are further complicated by the fact that the **moon's orbit is elliptical,** with the earth positioned at one of the foci.

↳ The moon travels faster at perigee than at apogee (Kepler's law of equal area) so this also leads to variations in the 24h50m and 12h25m cycles.

↳ The difference in distance between perigee and apogee is 13%, which affects the amplitudes of the tides.

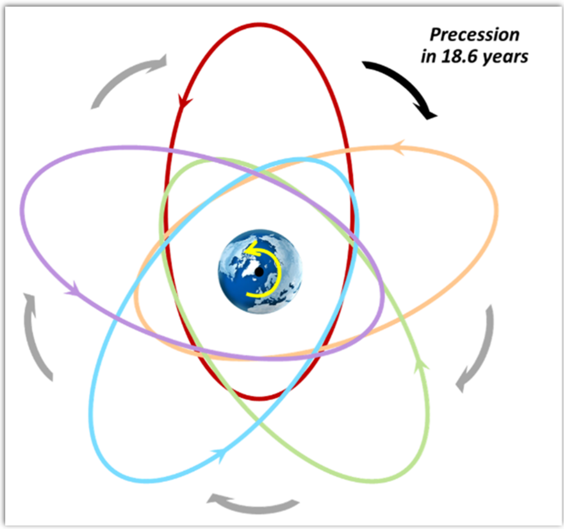


Moon’s elliptical orbit

⇨ Further modulation comes from the fact that the moon’s orbit is elliptical and exhibits precession on an 18.6 year cycle.

↳ This alters the alignment of the moon’s orbit compared to our orbit around the sun.

↳ This introduces a long-timescale to the variations of the tides (period and amplitude), which can be identified in long-term tidal records.

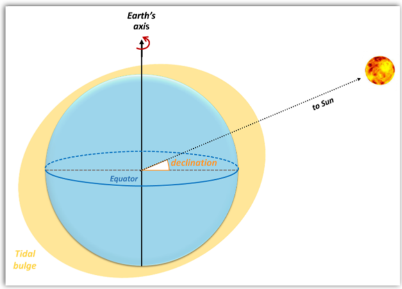


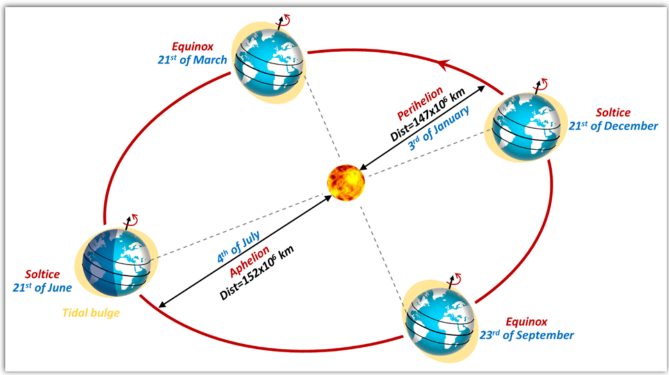
Solar tides

⇨ The sun also produces tides, but less than half as strong even though the sun’s gravitational attraction is much stronger.

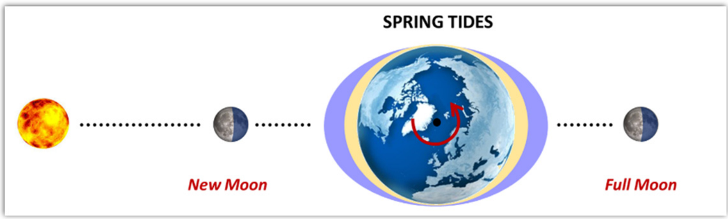
⇨ Earth's axial tilt in the plane of the ecliptic (23.4°), leading to an asymmetry between the two high tides of the day

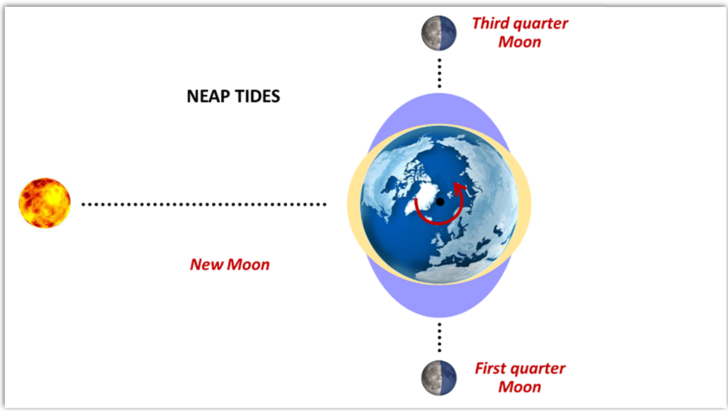
⇨ Our orbit around the sun is also elliptical with a 4% difference between perihelion and aphelion.





Spring tides and neap tides

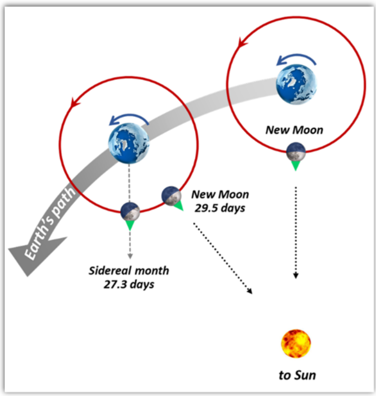




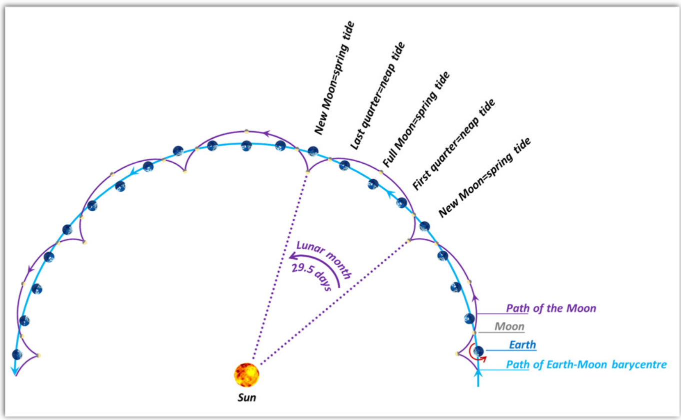
⇨ The sun and moon combine to modulate the amplitude of the tides.

↳ This is the main effect responsible for the semi-monthly modulation.

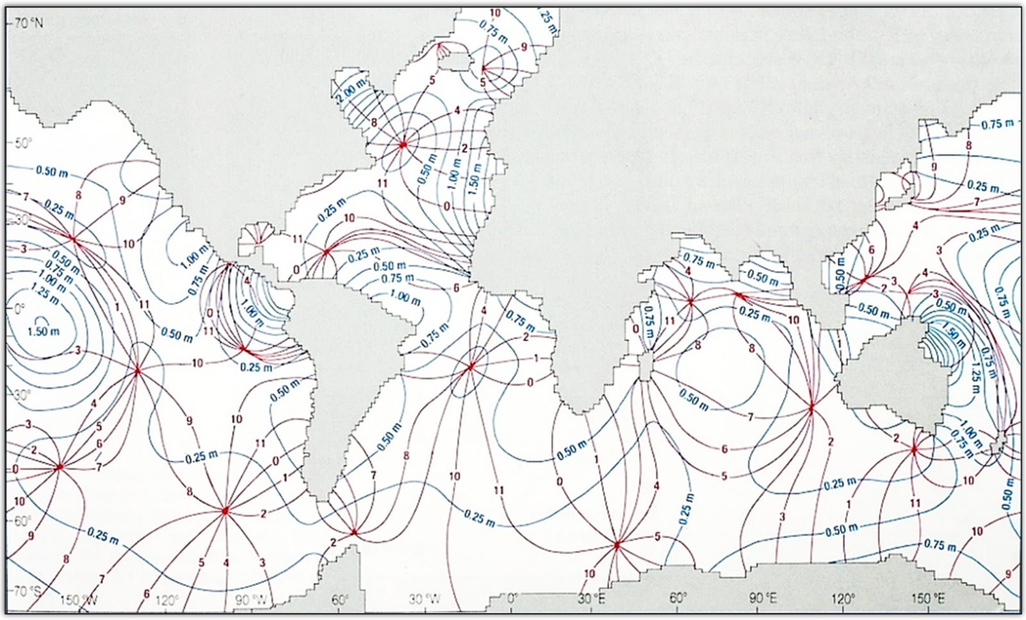
The lunar month



⇨ The lunar orbital period is actually 27.3 days but the lunar month (or "synodic month") is 29.5 days.



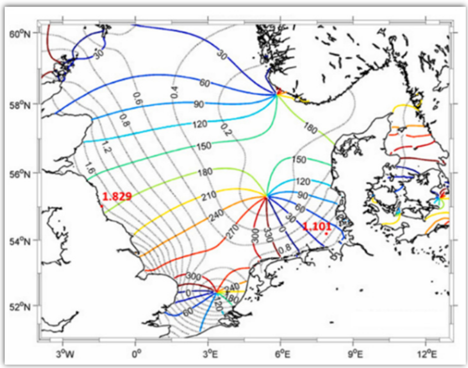
Amphidromic systems: M2 tide

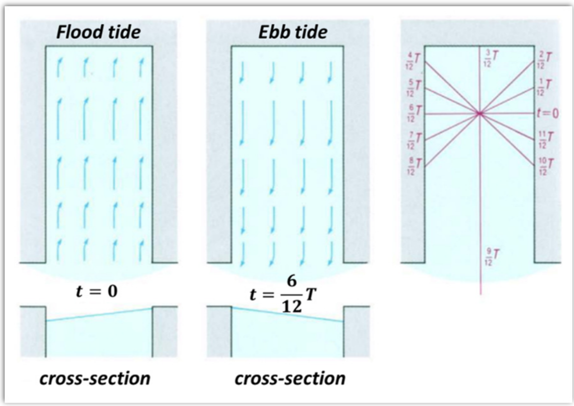


Coriolis deflection

⇨ The Coriolis force pushes to the right in the northern hemisphere and to the left in the southern hemisphere.

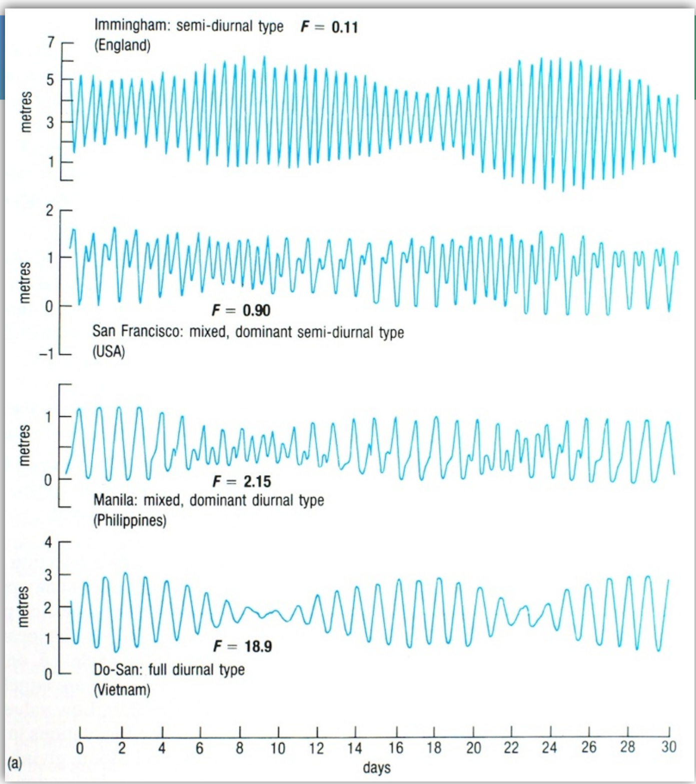
↳ This bunches up water against coasts and explains the counter-sense rotation of tides around nodal points.





Mixed tides

⇨ Combined Equatorial and Tropic tides and lunar and solar tides give very different tidal signatures in different locations



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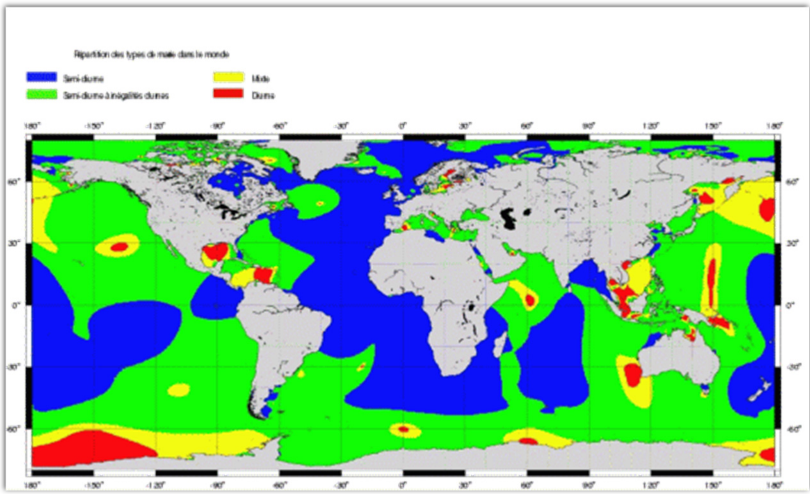
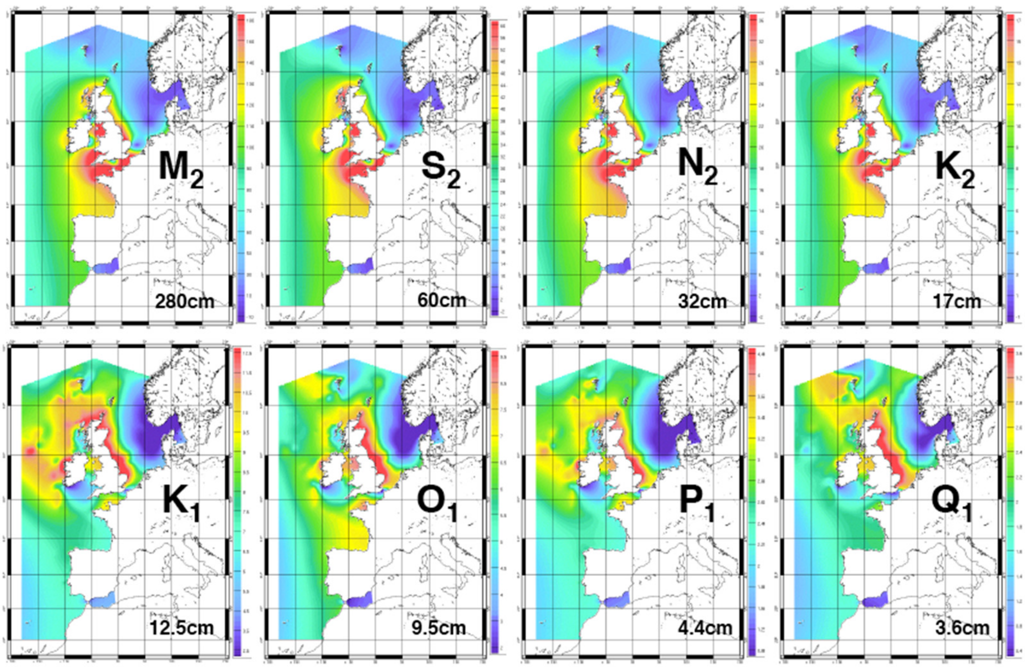


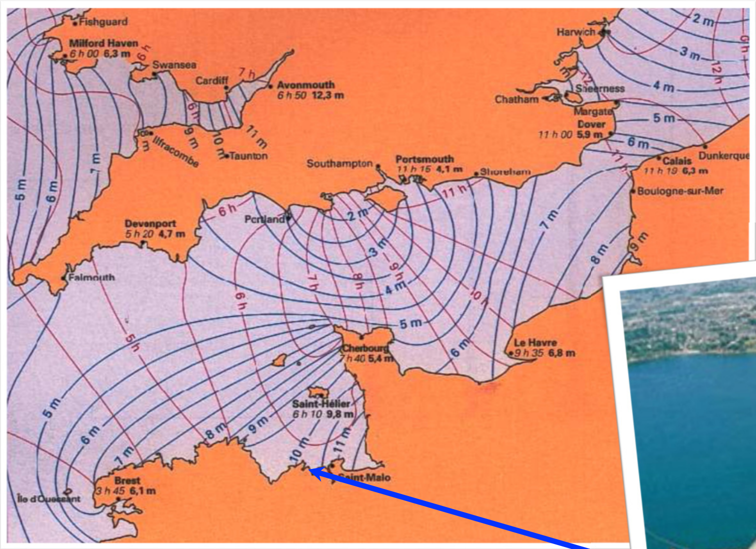
Fourier coefficients

Tidal components	Symbol	Periods (hours)	Coefficients ratio
Semi-diurnal :			
Principal Lunar	M_2	12.42	100
Principal Solar	S_2	12.00	46.6
Larger Lunar elliptic	N_2	12.66	19.2
Luni-solar	K_2	11.97	12.7
Diurnal :			
Luni-solar	K_1	23.93	58.4
Principal lunar	O_1	25.82	41.5
Principal solar	P_1	24.07	19.4
Longer Periods:			
Lunar fortnightly	M_f	327.86	17.2
Lunar monthly	M_m	661.30	9.1

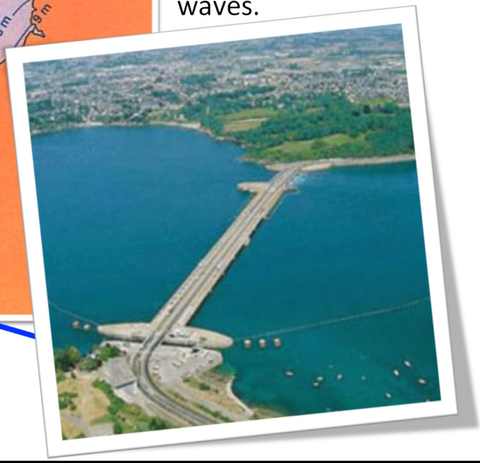
Around Europe



The tidal range



⇨ The amplitude of the tide can vary greatly around a coast, depending on propagation characteristics of coastally trapped Kelvin waves.



↳ Tidal power barrages are installed in regions where the amplitude is high.