Chapter1: General Properties of the Waves

- *Wind-forced waves*
- *Observed spectrum*
- *Wave kinematics*
- *Wavenumber and frequency*
- *Phase and group speed*
- *Dispersion relation*

Wind-forced waves (1)

⇨ Here is a **hypothesis** to explain how ocean waves might grow:

❶ Let's just imagine the surface of the ocean. It is almost flat with a few undulations to it and a wind is blowing across.

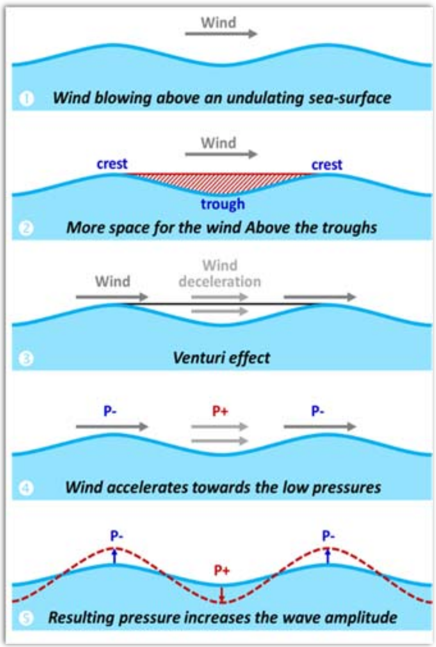
❷ Near the sea-surface, there is a little bit more space above the trough than above the crest (red striped area).

❸ So, the wind is going to occupy more space above the troughs and by conservation of mass, it will slow down. This is the **Venturi effect**.

❹ **Bernoulli's theorem** indicates that changes in the wind speed are associated with a **pressure gradient force**. The wind accelerates from high (**P+**) to low (**P-**) pressures and it decelerates towards higher pressures (**P+**).

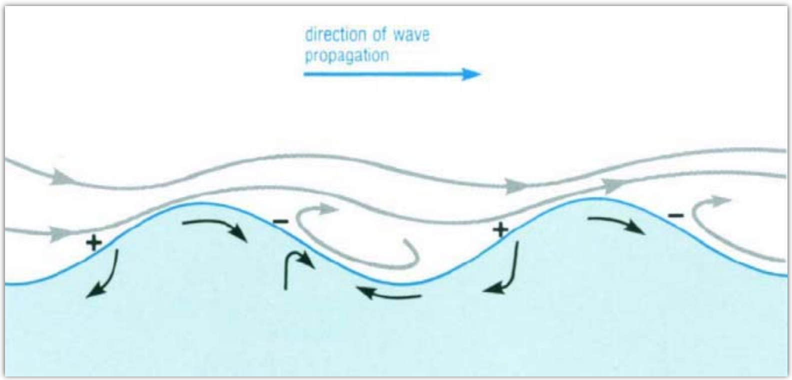
❺ As a consequence, relative to the average pressure, there is slightly less pressure (**P-**) above the crests and slightly more (**P+**) above the troughs. The ocean surface will thus be **pushed up** at the crests and **pushed down** in the troughs, increasing the wave amplitude.

↳ The wind blowing over a slight undulation makes the undulation get slightly bigger.



Wind-forced waves (2)

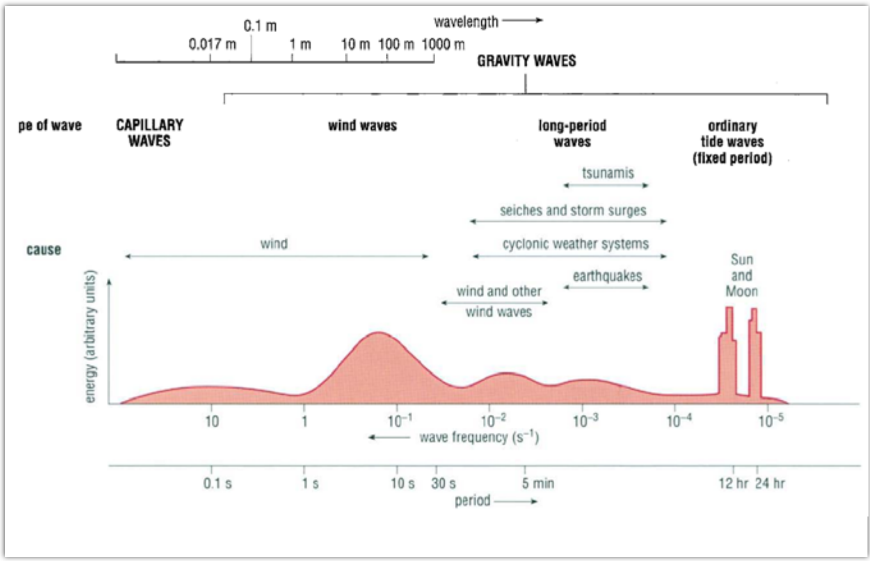
⇨ The “sheltering” model of wave forcing.
The presence of waves modifies the air flow - creates pressure differences that serve to push the wave.
Applies to situations where winds of > 1m/s and faster than wave speed.
Waves also need to be steep enough for the effect to work.



[Wave, Tides and shallow-water processes, Open university]

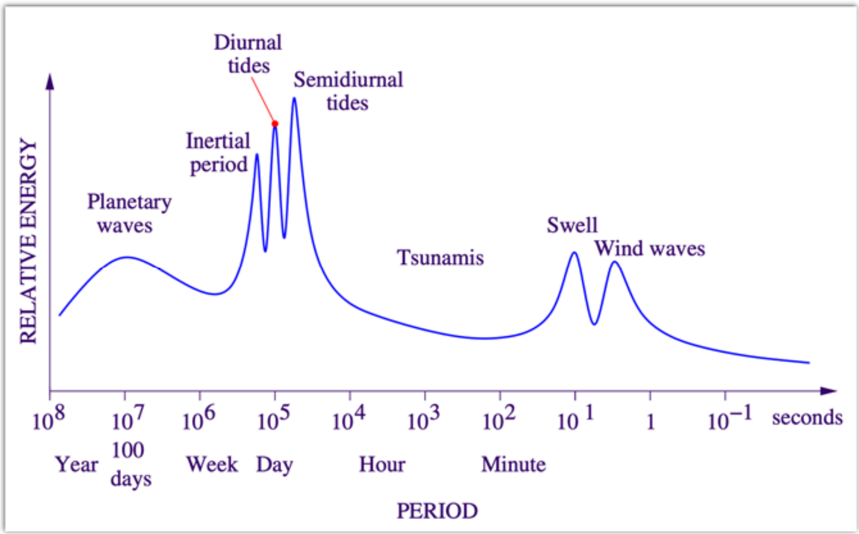
Observed spectrum (1)

⇨ Distribution of frequency for sea waves, and some of their causes.
(note that for some frequencies the energy may have been transferred from other frequencies)



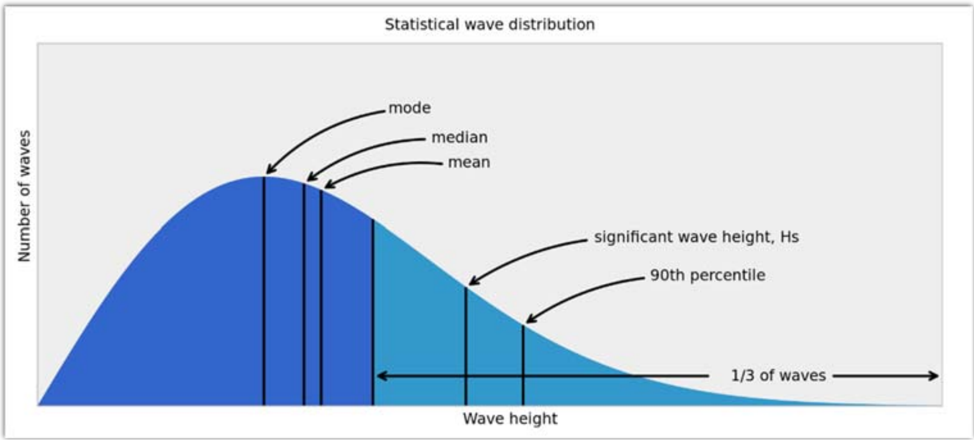
[Wave, Tides and shallow-water processes, Open university]

Observed spectrum (2)

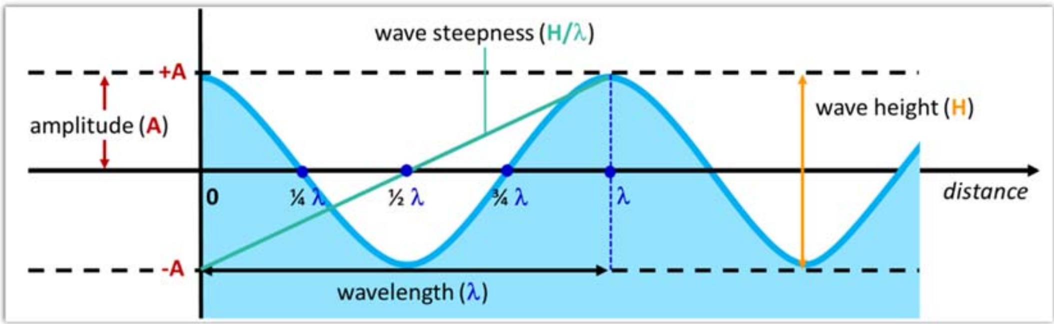


[Pond and Pickard, 1983, p209]

Wave height distribution

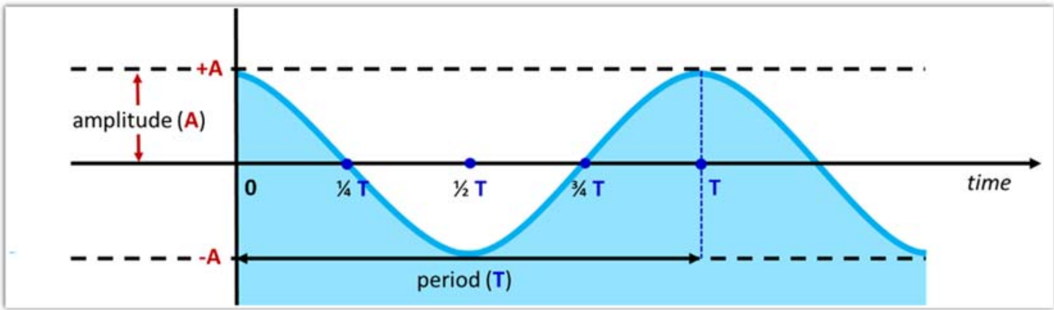


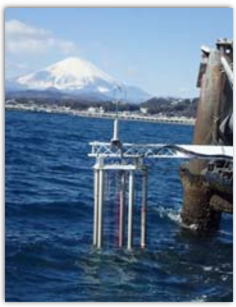
Basic properties in space



Wave propagating to the right (x increases):
- Amplitude (**A**)
- Wave height (**H**)
- Wavelenght (**λ**)
- Steepness (**H/λ**)

Basic properties in time





- Amplitude (**A**)
- Wave height (**H**)
- Period (**T**)

⇨ As well as wavelength and period we also represent waves in terms of frequency and wavenumber

Wave kinematics (1D in space)

⇨ Consider a propagating sinusoidal wave:

$$\eta(x,t) = \boldsymbol{A}\cos(kx-\omega t+\phi)$$

⇨ Equivalently: $\eta = Re\, \tilde{\boldsymbol{A}}\, e^{i(kx-\omega t)}$

⇨ Equivalently: $\eta = Re\, \tilde{\boldsymbol{A}}\, e^{i\theta}$

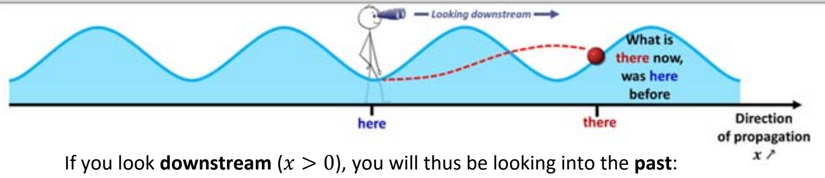
How many parameters do we need to describe this wave?

⇨ Equivalently: $\eta = Re\, \boldsymbol{A}\, e^{i\theta} e^{i\phi}$

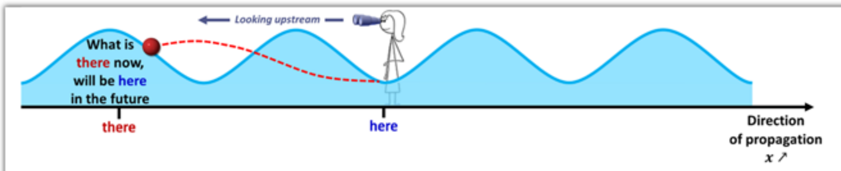
⇨ Equivalently: $\eta = \boldsymbol{A_1}\cos(kx-\omega t) + \boldsymbol{A_2}\sin(kx-\omega t)$

Wave kinematics (1D in space)

Why is there a minus in front of ω?



If you look **downstream** ($x > 0$), you will thus be looking into the **past**:
*What is **there** ($x > 0$) now, <u>was **here** before</u>* ($t < 0$).



If you look **upstream** ($x <$), you are looking at oscillations that will arrive at your position in the **future**: *What is over **there** now* ($x < 0$), <u>*will be **here** in the future*</u> ($t > 0$).

Wave kinematics (2D in space)

⇨ Consider a propagating sinusoidal wave:

How many parameters do we need to describe this 2D wave?

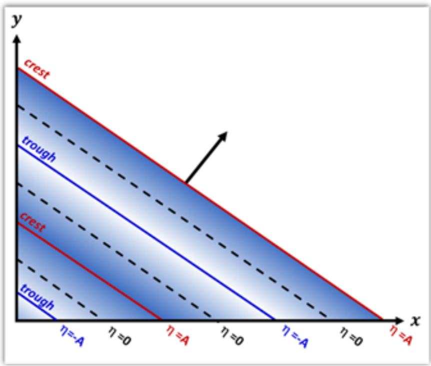
$$\eta(x,y,t) = \boldsymbol{A}\cos(lx + my - \omega t + \phi)$$

⇨ Equivalently: $\eta = Re\,\tilde{\boldsymbol{A}}\,e^{i(\boldsymbol{k}\boldsymbol{x}-\omega t)}$

$$\boldsymbol{k} = \binom{l}{m} \qquad \boldsymbol{x} = \binom{x}{y}$$

⇨ Equivalently: $\eta = Re\,\tilde{\boldsymbol{A}}\,e^{i\theta}$

⇨ Equivalently: $\eta = Re\,\boldsymbol{A}\,e^{i\theta}e^{i\phi}$



⇨ Equivalently:

$$\eta = \boldsymbol{A_1}\cos(kx + my - \omega t) + \boldsymbol{A_2}\sin(kx + my - \omega t)$$

⇨ We note that the wavenumber has the properties of a vector

Period, wavelength and speed

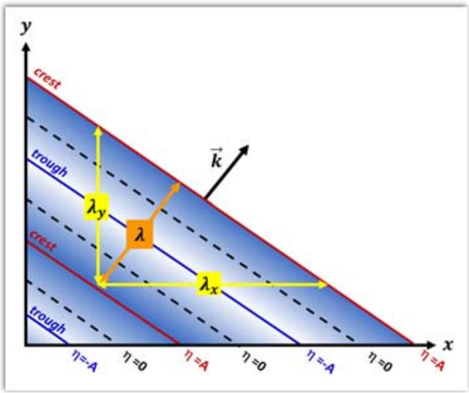
⇨ Wavelength depends on direction

$$\lambda_x = \frac{2\pi}{l} \quad \lambda_y = \frac{2\pi}{m} \qquad \lambda = \frac{2\pi}{k}, \qquad k^2 = l^2 + m^2, \qquad \frac{1}{\lambda^2} = \frac{1}{\lambda_x^2} + \frac{1}{\lambda_y^2}$$

↳ Wavelength is *not* a vector

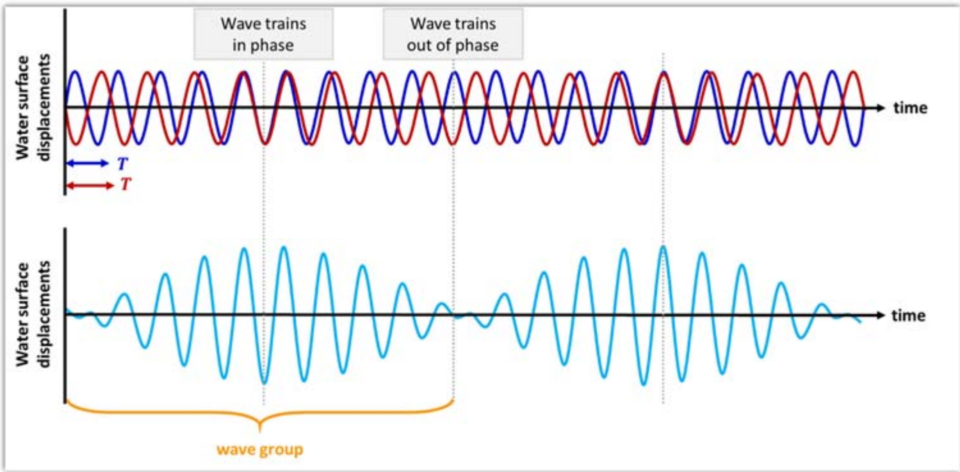
- Period $T = \dfrac{2\pi}{\omega}$
- Phase speed $c_x = \dfrac{\omega}{l} \qquad c = \dfrac{\omega}{k}$

(not a vector either)



Interference and modulation

⇨ Interference between frequencies leads to a **modulation envelope** that travels at a different speed to the individual waves.

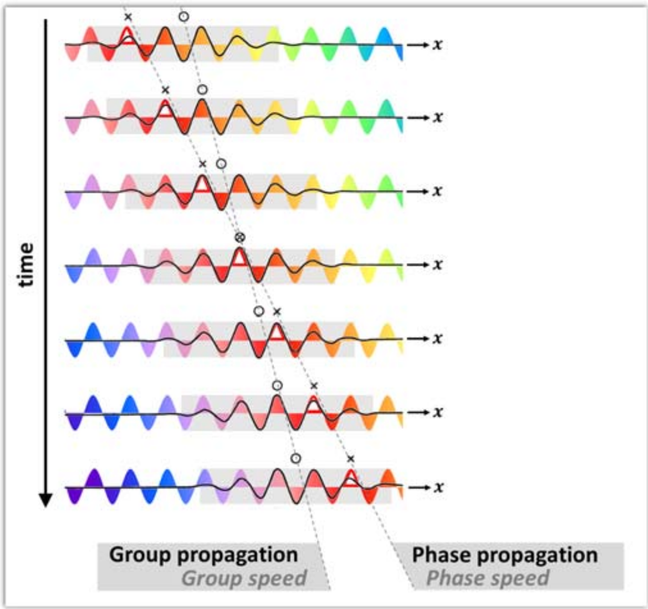


Group propagation

⇨ For normal group propagation, the wave packet moves at half the speed of the phase propagation.

⇨ To study the relative speeds of the packet and the wave crests, we need to find the "dispersion relation"

⇨ For this we need to move beyond kinematics and involve the physical properties of the medium.



Group propagation

⇨ Consider 2 waves of similar frequency and wavenumber

$$\eta = Re\,\tilde{A}\left[e^{i(k_1x-\omega_1t)} + e^{i(k_2x-\omega_2t)}\right]$$

$$= Re\,\tilde{A}\,e^{i(kx-\omega t)}\left[e^{i(\Delta kx-\Delta\omega t)} + e^{-i(\Delta kx-\Delta\omega t)}\right]$$

$$\begin{cases} k_1 = k+\Delta k, & k_2 = k-\Delta k \\ \omega_1 = \omega+\Delta\omega, & \omega_2 = \omega-\Delta\omega \end{cases}$$

$$\eta = \underbrace{Re\,\tilde{A}\,e^{i(kx-\omega t)}}_{\text{Rapidly varying signal / Original signal}} \times \underbrace{2\cos(\Delta kx-\Delta\omega t)}_{\text{Slowly varying envelope / Modulation}}$$

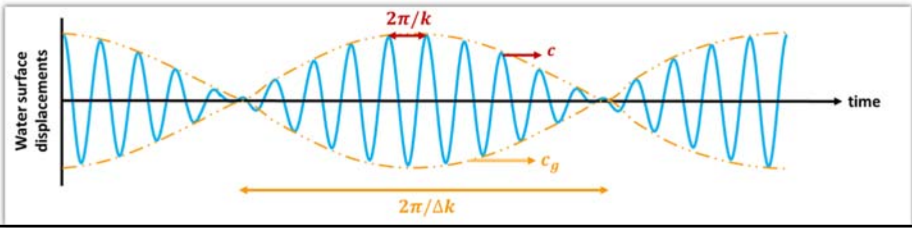
Rapidly varying signal
Original signal

Slowly varying envelope
Modulation

➢ **Envelope** moves with speed

$$\lim_{\Delta\to 0}\frac{\Delta\omega}{\Delta k} = \frac{\partial\omega}{\partial k}$$

➢ **Group velocity** $\mathbf{c}_g = \nabla_k\omega$
= rate of propagation of information or energy



Dispersion

⇨ However....

$\frac{\omega}{k}$ and $\frac{\partial\omega}{\partial k}$ are not necessarily the same for all values of *k*.

.... so it is possible that the envelope will distort as it displaces.

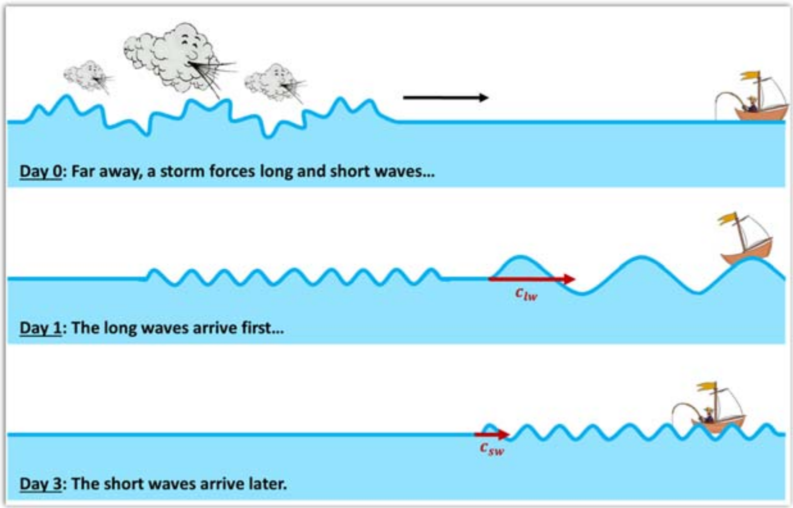
This is called **dispersion**.

3 cases:
1) Long-waves go faster than short-waves
2) Short-waves go faster than long waves
3) Short and lon waves propagate at the same speed

Dispersion

CASE1: short waves go slow, and long waves go fast.

↳ *What would happen to a wave that is a mixture of short waves and long waves?*



Dispersion relation/ Dispersion diagram

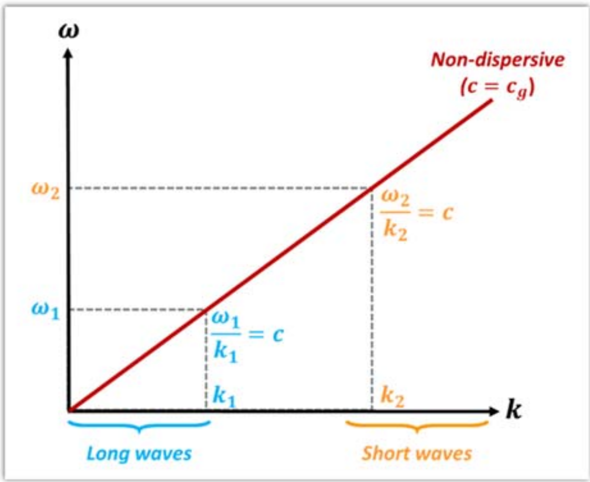
⇨ The relationship between ω and k is called the dispersion relation.

⇨ If this relationship is linear (and of course ω=0 when k=0),

$$\frac{\partial\omega}{\partial k} = \frac{\omega}{k}, \quad c_g = c$$

↳ the wave is "non-dispersive"

For non-dispersive waves, all the wavelengths propagate at the same speed. A wave pattern (sum of different wavelength) will not change its shape during its propagation

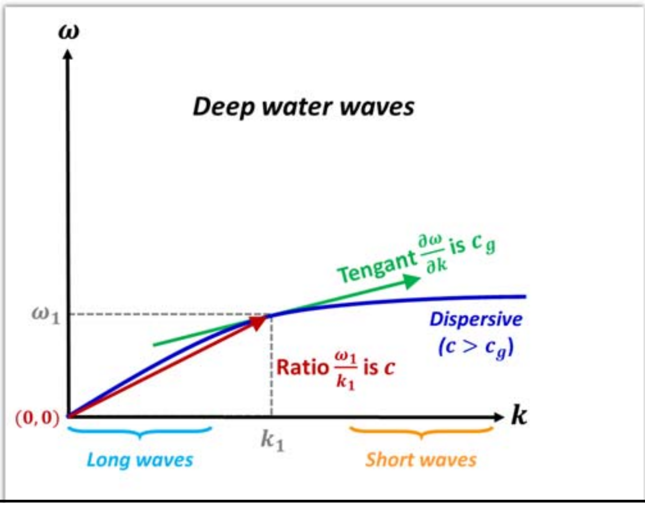


Dispersion diagram

⇨ The relationship between ω and k is called the dispersion relation.

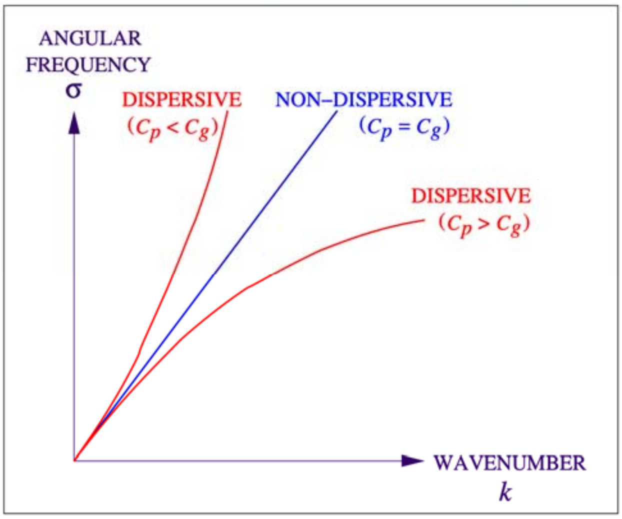
⇢ **the phase speed** (c) is the arrow that points from the origin toward the curve (the ratio $\frac{\omega}{k}$)

⇢ **the group speed** (c_g) is the tangent to the curve ($\frac{\partial\omega}{\partial k}$)



Dispersion diagram

Phase Velocity and Group Velocity



Phase velocity, $c_p = \dfrac{\sigma}{k}$; Group velocity, $c_g = \dfrac{\partial\sigma}{\partial k}$