CHAPTER 5
Tides

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WAVES5 Contents

This chapter provides a **descriptive introduction** to **tides**, tidal processes, and tidal forces.

Tides are gravity waves characterized by the rhythmic rise and fall of the sea level. Tidal flow and ebb at the coast is a manifestation of the general rise and fall of sea level caused by a long-wavelength wave motion that affects the open ocean as well as shallow coastal waters. Because of their long periods and long wavelengths, tidal waves behave like **shallow-water waves**. Horizontal tidal currents are the horizontal movements associated with the rise and fall of these shallow-water waves (see **#WAVES2.2h**).

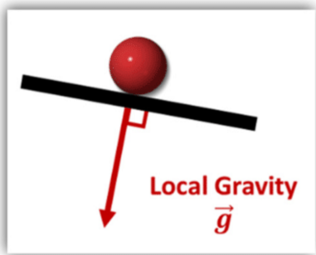
Sea level varies around a reference point. In an introductory section (see **#WAVES5.1**), we will discuss the shape of the earth and the **shape of the mean sea level** called the geoid. We will identify different **tidal frequencies** in **observed sea level measurements**. We will then consider the forces that cause the tides. These are the **gravitational forces** exerted by the **moon** and the **sun** upon the earth and the oceans. To derive the Tide-Producing Force, we will express the effects of the gravitational attraction of a body orbiting the Earth (see **#WAVES5.2**). In sections **#WAVES5.3** and **#WAVES5.4**, we will thus examine the **relative motions** of the earth, moon, and sun that give rise to tractive forces that displace the oceans and yield complex patterns of tidal events. In **#WAVES5.5**, we will give a more realistic description of how the tides vary around the world including the effect of the **Coriolis force**.

🕮 Note that the Atmosphere is also subject to tides, as is the solid Earth to a lesser extent. But because the Earth is denser and more viscous than water or air, before it can react to the tidal forces, the forces have already changed. So, the crustal tides are very small compared to the movements in the ocean. Seawater is free to move and responds quickly to tidal forces with fluctuations in sea level.

WAVES5.1: Introduction

5.1.a) The equilibrium sea surface

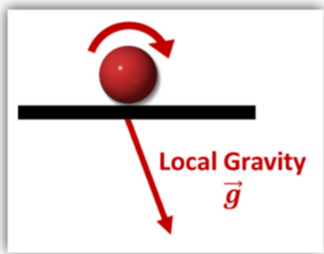
⇨ Let's discuss the shape of the earth and ask *What is the definition of a **horizontal surface**?*



↳ A horizontal surface is a surface where if you put a ball on it, the ball will stay put and not roll one way or the other. Conversely, if the surface is not horizontal, the ball will roll down.

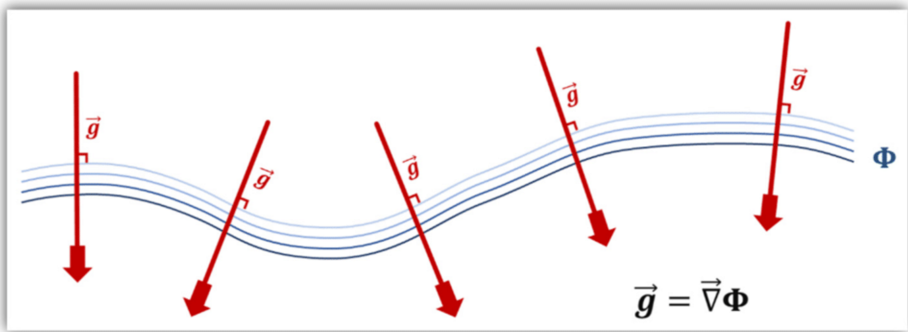
⇨ So, the definition of a **horizontal surface** is related to gravity.

It is the surface to which the local gravitational acceleration is perpendicular.



✋ Because the earth is not a perfect sphere, the local horizontal surface is not necessarily the plane that is perpendicular to a line pointing towards the centre of the earth.

⇨ If we consider the **gravitational potential**, so that the acceleration of gravity is at right angles to equipotential surfaces, then a horizontal surface is a surface of equal gravitational potential. The equipotential surface that coincides with the mean sea level is called the **geoid**.



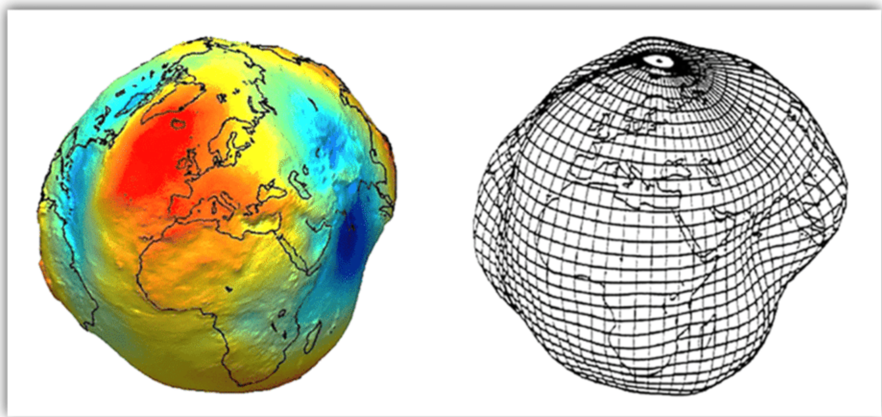
5.1.b) The geoid

⇨ The shape of the earth is not a sphere. The earth is spinning so that it has an **oblate spheroid** shape. But it is actually more complicated because of geology and continents.

↳ Suppose that there is some excess mass (high density) buried under the ocean floor. This will distort the equipotential surface so that it bulges upward, resulting in a small rise in sea level above excess mass. Similarly, if there is a mass deficit (low density), there will be a slight depression in the sea surface.

⇨ So the actual shape of the earth is thus quite complex. It looks like a potato. However, in the representation below, all departures from sphericity have been greatly exaggerated.

↳ There is a large depression in the Indian Ocean ($\sim -100\ m$) and a bulge over the North Atlantic Ocean ($\sim +100\ m$).



⇨ Note that the shape of the geoid changes on geological time scales. Mountain building and continental drift take ~millions of years. On shorter time scales, glacial effects can modify the shape of the geoid. For instance, due to the melting of the Laurentide Ice Sheet after the last Ice Age (10,000 years ago), Canada is still rebounding from the lifted weight and moving away from the centre of the earth.

↳ But for the purpose of the study of daily/monthly fluctuations, we can assume that this is the shape of the earth and estimate the variations in sea level relative to it.

⇨ All departures of the oceanic surface from the geoid are associated with either tides, waves, or ocean currents. Note that some of the departures from the geoid are actually quite steady in time compared to the time scales of the tides. In particular, sea level gradients that translate into horizontal pressure gradients can be balanced by the Coriolis force. This gives rise to relatively permanent currents such as the Gulf Stream.

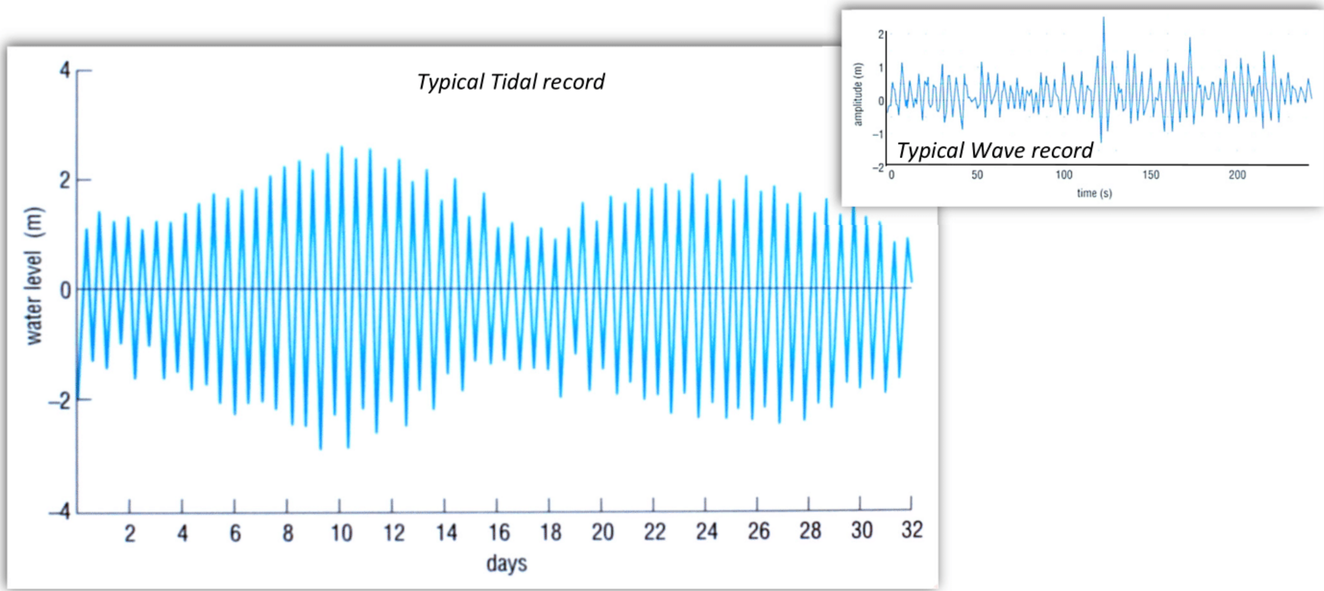
↳ The sea level fluctuations associated with tides must be estimated relative to this mean sea level. Let's have a look at a measurement of sea level variations associated with tides.

5.1.c) A typical tide gauge reading

⇨ Here is a typical 30-day tidal record showing water level oscillations. It comes from a coastal station in the Tay estuary in England.

⇢ We notice that the tidal fluctuations of sea level are of the same order as the oscillations of surface wind-generated waves (a few meters), but their period is substantially longer. In comparison, wind-wave oscillations are quite irregular (both in amplitude and period), with typical periods of the order of seconds to a few tens of seconds (see **#WAVES1.1b**).

⇢ Tidal fluctuations are very regular compared to wind waves. We estimate a period of about 12 hours, i.e. high and low tides occur twice a day. This is the **semi-diurnal tide**.



⇢ The amplitude of the semi-diurnal tide also varies throughout the month. Between day 6 and day 14, the amplitude is quite high, while between day 14 and day 20 the amplitude is lower. We observe that this modulation envelope undergoes two maxima and two minima of within a month.

↳ This time series is quite similar to the interference between two waves of similar frequency, resulting in a lower frequency modulating the amplitude of a faster signal (see **#WAVES1.3a**). There are indeed two tidal frequencies, both close to the semi-diurnal frequency, but not exactly the same. We will see that the moon and the sun both contribute to the tides at slightly different frequencies, which explains this modulation pattern in the tidal record.

➢➢ In the next sections, we will discuss why the forces (see **#WAVES5.2**) associated with the relative motions of the moon and the earth (see **#WAVES5.3**), and the sun and the earth (see **#WAVES5.4a**) cause oscillations with semi-diurnal periods.

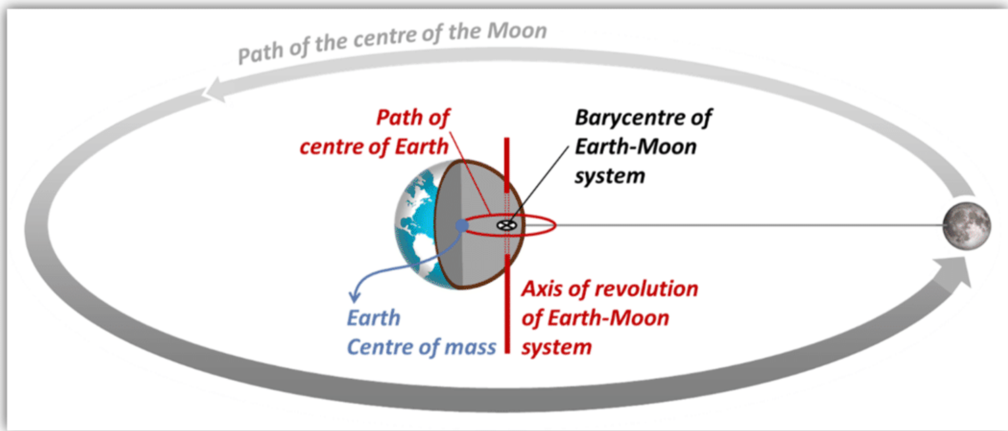
WAVES5.2: Tidal Forces and Semi-Diurnal Periods

5.2.a) Moon's orbital motion

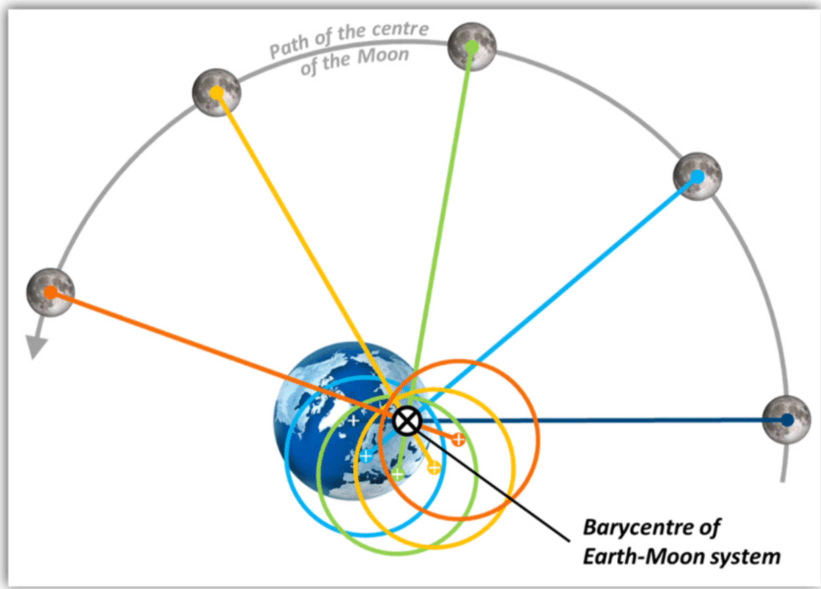
⇨ It is common knowledge that the moon orbits the earth once every 27.3 days (a sidereal month). EXCEPT **this is not quite true**.

↳ *What would happen if the earth and the moon were of the same size? Would the moon go around the earth or would the earth go around the moon?* They would dance around each other, around their common centre of mass which would be halfway between the two bodies.

↳ The moon does not simply revolve around the centre of the earth. The **earth and the moon behave as a single system**, rotating around their common centre of mass. Since the earth is a hundred times more massive than the moon, their **barycentre actually lies inside the earth**, at 4700 km from its centre (radius of the earth is $a = 6400km$).



⇨ This means that as the moon goes around the earth in 27.3 days, the earth also **revolves eccentrically** about the centre of gravity of the Earth-Moon system.

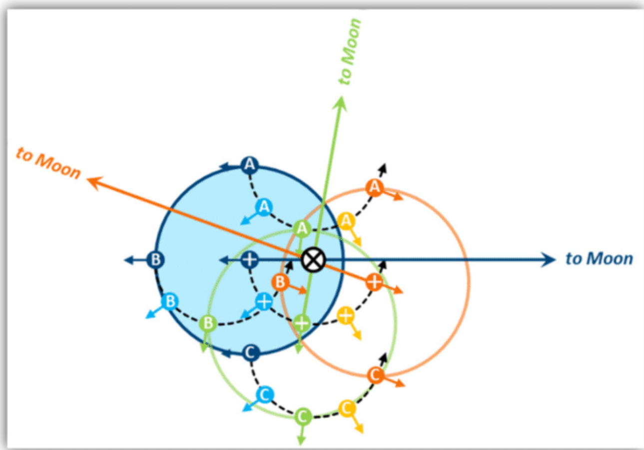


✋ This eccentric motion has nothing to do with the earth rotation (spin) upon its axis in ~23 hours, 56 minutes, and 4 seconds.

5.2.b) The Tide-Producing Force

⇨ Now let's consider the forces acting on a fluid on the surface of the earth due to the rotation of the Earth-Moon system and due to the gravitational force exerted by the moon.

- **<u>The centrifugal force</u>** acting on the Earth-Moon system exactly balances the forces of gravitational attraction between the two bodies, so that the Earth-Moon system remains in equilibrium, i.e., the moon does not escape from the earth or fall toward the earth.



⇢ In an absolute frame, the earth **translates** in a circuit around the barycentre of the Earth-Moon system, so that any point (+ A B C) inside or on the surface of the earth describes **the same trajectory**: an anticlockwise arc in space with **the same curvature**. ✋ The <u>earth is not pivoting</u> to present the same face to the moon.

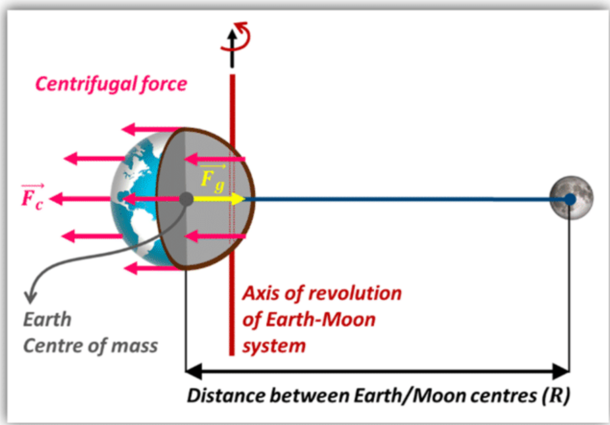
↪ So the **centrifugal force has the** same strength **everywhere**, at all points inside or on the surface of the earth, **pointing away from the moon**. ✋ Adding the earth's rapid rotation does not alter this.

⇢ The **magnitude of the** uniform **centrifugal force** is equal to the gravitational force between the earth and the moon. It is given by the law of gravity:

$$|Fc| = \frac{GM_1M_2}{R^2}$$

with:

- G the universal gravitational constant ($G = 6.672 \times 10^{-11}\ N\ m^2\ kg^{-2}$)
- M_1 the mass of the earth ($M_1 = 5.972 \times 10^{24}\ kg$)
- M_2 the mass of the moon ($M_2 = 7.348 \times 10^{22}\ kg$)
- R the distance between the centres of the earth and the moon ($R = 384\,400\ km$).



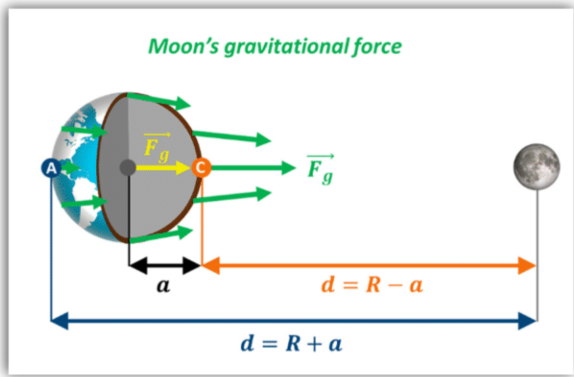
▪ **The magnitude of the gravitational force** produced by the moon upon the earth is not the same at different positions on the earth because the magnitude of the gravitational force exerted **varies with the distance** to the attracting body. ***And therein lies the subtilty***.

$$|Fg| = \frac{GM_1M_2}{d^2}$$

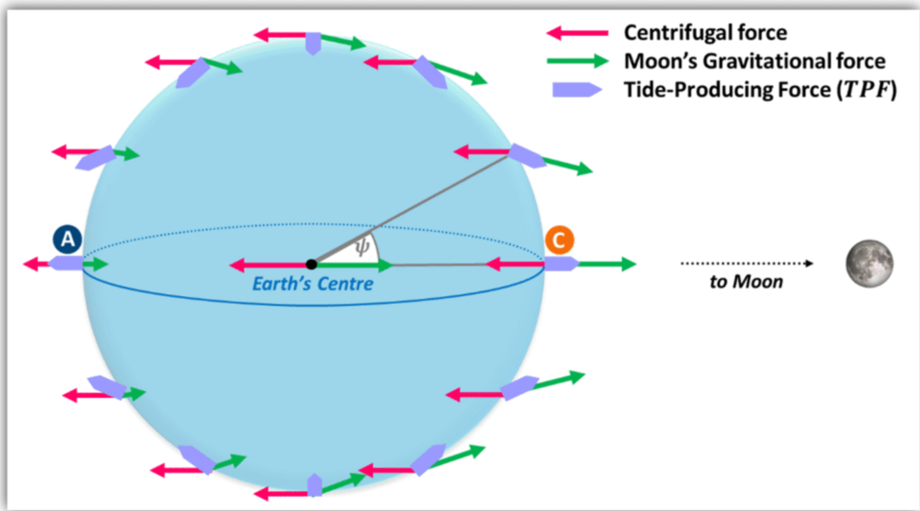
▪ with d the distance between a point anywhere on Earth and the centre of gravity of the moon.

⇢ Points nearest the moon (C) experience a greater gravitational pull from the moon than those on the opposite side of the Earth (A).

⇢ The direction of the moon's gravitational pull is toward the centre of mass of the moon. It is not exactly parallel to the direction of the centrifugal force, except along the line connecting the centres of the Earth and the Moon.



▪ The **Tide-Producing Force** (***TPF***) at the earth's surface results from the difference between these two basic forces: the uniform **centrifugal force** produced by the revolutions of the earth and moon around their common centre of gravity (in red), and the varying **gravitational force** exerted by the moon at the earth surface (in green).



⇢ The gravitational force exerted by the moon at the earth's centre (green) is exactly equal and opposite to the centrifugal force there (red), so the net force **at the centre of the earth is zero**.

⇢ **At point A**, further away from the moon ($d = R + a$), the gravitational force is slightly weaker, so that the net force, the Tide-Producing Force (TPF) is:

$$TPF_A = \frac{GM_1M_2}{(R+a)^2} - \frac{GM_1M_2}{R^2}$$ with a the radius of the earth

↪ At point A, gravity loses and the net force (purple arrow) is directed opposite to the moon.

⇢ On the opposite side of the earth, **at point C**, closer to the moon, the net force is towards the moon because the centrifugal force is the same but the gravity is slightly stronger:

$$TPF_C = \frac{GM_1M_2}{(R-a)^2} - \frac{GM_1M_2}{R^2} = \frac{GM_1M_2a(2R-a)}{R^2(R-a)^2}$$

↳ On the side facing the moon C, the ocean is attracted to the moon, whist on the side opposite to the moon A, the ocean is pushed in the opposite direction to the moon (~stretching effect).

⇨ Acknowledging that the radius of the earth is small compared to the distance between the earth and the moon ($a \ll R$), the magnitude of the **Tide-Producing Force** at any point on the surface of the earth can be simplified:

$$|TPF| \approx \frac{GM_1M_2 2a}{R^3}$$

The forces that trigger the tide (centrifugal and gravitational forces) follow $1/R^2$ laws, while the Tide-Producing Force is proportional to $1/R^3$.

While the sun's gravitational pull of is stronger than the moon's, **the moon has a stronger tidal effect than the sun.** This is because the moon is so much closer to the earth (~380,000 km) than the sun (~150,000,000 km). The moon is actually twice as important as the sun in generating tides (see **#WAVES5.4a**).

⇨ The magnitude of the vertical component of the Tide-Producing Force is tiny compared to the local gravitational force on Earth ($\sim 9 \times 10^6$ smaller). It cannot actually create a tide. On the other hand, it is the resulting **horizontal component**, which does not have to compete with the earth's gravity, that is crucial for the forcing of tides.

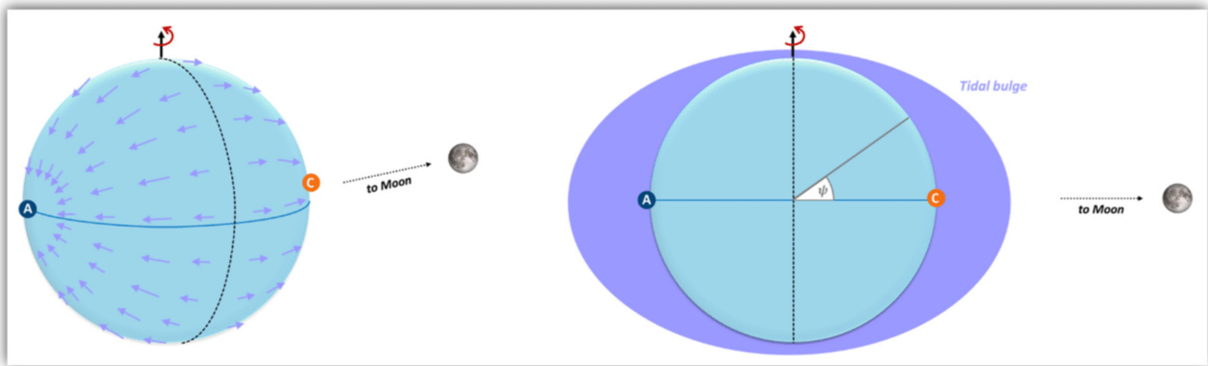
The moon is not that far from the Earth ($380{,}000$ kilometres, i.e. barely a 3-day trip in a spaceship – most of us probably drive that distance in our lifetimes quite easily). It is close enough that the moon's gravitational pull is not parallel to the direction of the centrifugal force. The resulting **horizontal force** depends on the position of the earth relative to the position of the moon (the angle $\boldsymbol{\Psi}$). It is called the **tractive force**. The effect of this force can accumulate over large horizontal distances to cause significant changes in sea level.

⇨ The tractive forces on both sides, facing the moon or opposite to the moon, are **equal and opposite**. This portrays **converging horizontal components** that can cause the ocean to flow sideways and create a bulge.

5.2.c) Equilibrium tide and Earth rotation

⇨ The effect of the horizontal component of the Tide-Producing Force would be to move the water towards points A and C. They will displace the ocean to a new equilibrium shape: the equilibrium tide. This is represented below for a **spherical earth covered in ocean**.

The equilibrium tide has an ellipsoid shape, with two bulges pointing towards and away from the moon.

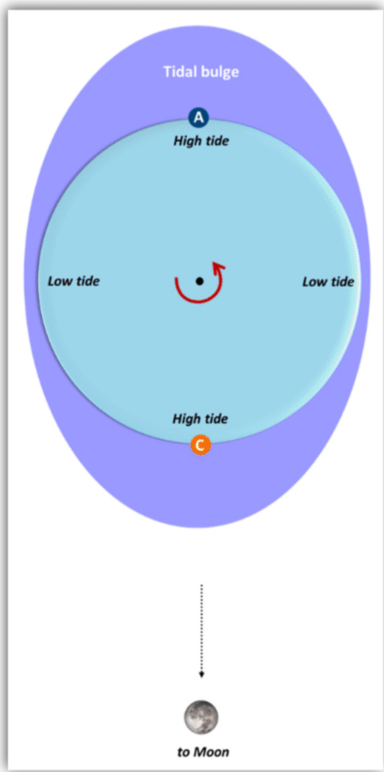


⇨ During the earth's daily rotation, any point on the surface would spin inside a prolate spheroid, just like a football (⚽) inside the a rugby ball (🏈. The point will thus experience **two high tides and two low tides** per day: the **semi-diurnal tide**.

The **wavelength** (λ) of the tidal wave is therefore half of the circumference of the earth.

- At the equator ($R \approx 6{,}400km$), the wavelength is $R \times \pi \approx 20{,}000km$.
- At higher latitudes, the distance around the earth is less and so is the tidal wave **wavelength**.

WAVES5.3: Lunar Tide: Periods and Amplitudes



5.3.a) Tidal Period

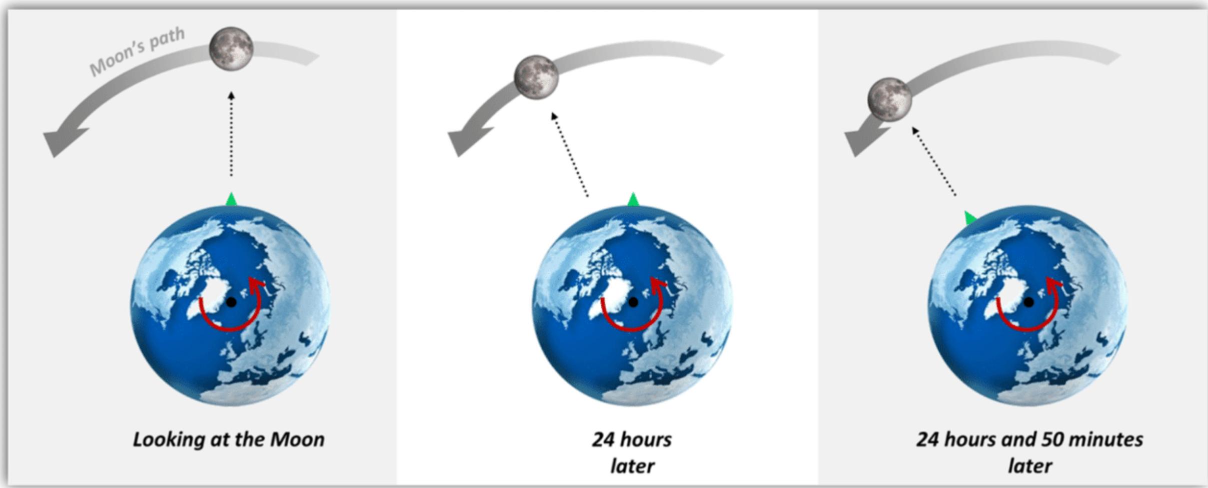
⇨ The semi-diurnal **equilibrium tide** arises from the earth spinning inside a symmetric force field associated with the moon's tractive force (see **#WAVES5.2**). The earth rotates within the bulge, producing two tides per day.

EXCEPT that the moon is not stationary. During the earth's daily rotation, the moon also moves.

⇢ The moon revolves about the Earth-Moon barycentre once every 27.3 days, in the same direction as the earth spins upon its own axis in 23.934 hours. During a sidereal day (the period of the earth's rotation relative to the fixed stars), the moon travels by about 1/27 of its revolution.

⇢ Starting from a position facing the moon (green triangle below), it would take **another ~50 minutes** for this point to be facing the moon again the next day. This explains why the moon does not rise at the same time every day. It rises about an hour later each successive day.

↪ This means that the period of the earth's rotation with respect to the moon is 24h and 50 minutes. This is the lunar day.



↳ The interval between two successive high (or low) tides is ~12 hours and 25 minutes. This is the period of the lunar semi-diurnal tide.

⇨ During a lunar day, there are two crests / two troughs associated with the lunar semi-diurnal tides and the wavelength of the equilibrium tide is half of the earth's circumference (see **#WAVES5.2c**).

⇨ In practice, the **equilibrium tide is never exactly realised** because as **Earth spins** about its own axis, the direction of the tractive force changes too quickly for ocean hydrodynamics to follow.

↳ At the equator, the wavelength is $\lambda \sim 20{,}000 km$. Given that the average depth of the ocean basins is less than $5000m$ ($= 5\ km$), i.e. much less than $1/20$ of the wavelength, **tidal waves are shallow-water waves** (see **#WAVES2.2g**).

↪ The phase speed of (barotropic) shallow-water waves is equal to $c = \sqrt{gH} \approx 221 m/s$.

↪ At the equator, the speed that is required to maintain the **equilibrium tide** is $c = \lambda/T = 20{,}000 km/12h25' \approx 435 m/s$.

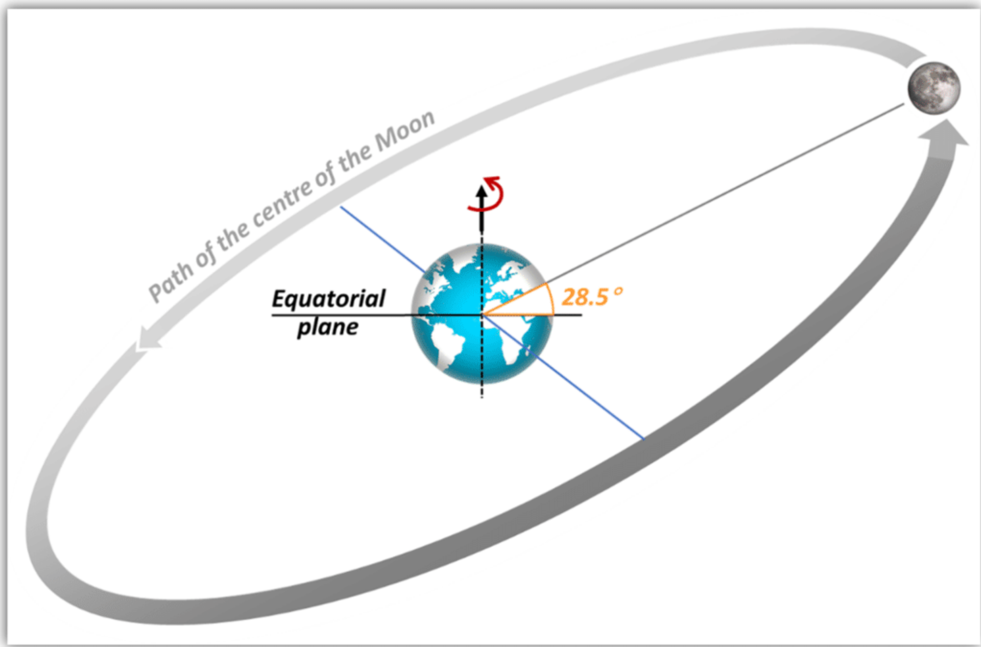
↳ At the equator, the tidal wave (the bulge) cannot keep up with the moon to maintain the **equilibrium tide**. The forcing frequency remains at 12h25' and in between the **tides are not in equilibrium**. **Equilibrium tides** could occur at higher latitudes, where the earth's circumference is less.

5.3.b) Tropic and Equatorial tides

⇨ The **lunar semi-diurnal tide** has a regular a forcing at **~12h and 25 minutes**.

EXCEPT that **the moon does not orbit in the earth's equatorial plane** (nor that of the sun).

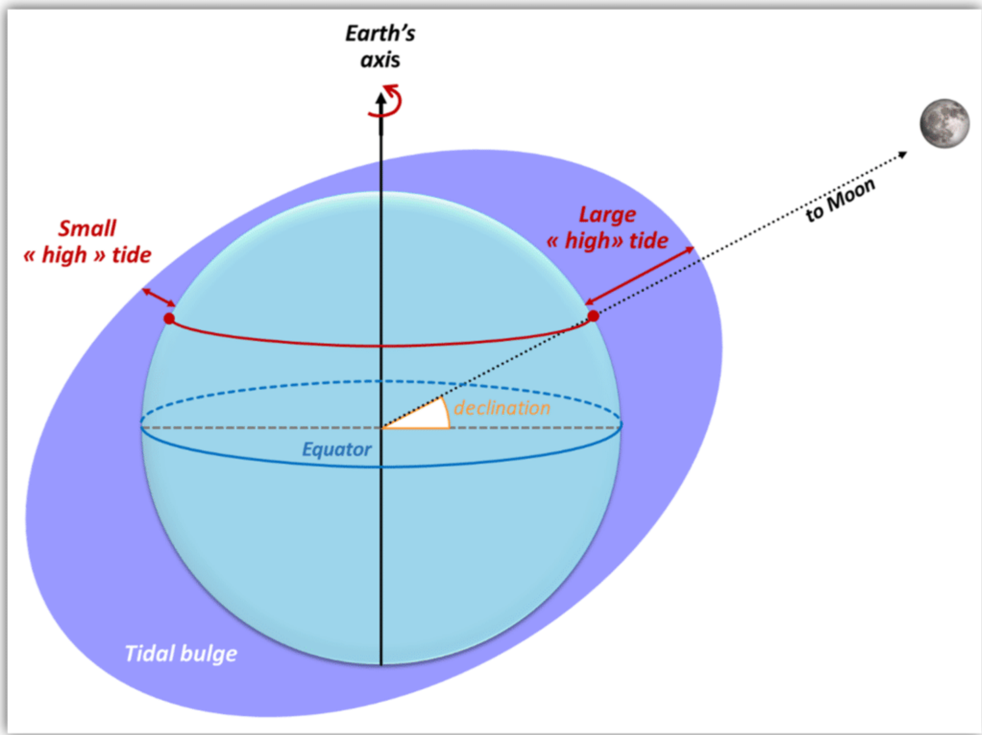
⇢ The moon's orbit is tilted by 28.5° with respect to the equator.



↪ This means that, during the 27.3-day period of the moon's rotation, a line connecting the centre of the earth to the centre of the moon makes an angle ranging from $-28.5°$ to $+28.5°$ to the equatorial plane. The **declination** angle of the moon is larger than the sun's declination (23.5°).

↪ Similar to the seasonal variation of the sun's path in the sky over a year, the successive paths of the moon in the sky appear to rise and fall over the monthly revolution of the moon.

↳ The tidal bulge is not lined up perpendicular to the earth's rotational axis as shown in the previous sections (although it does happen twice a month).



⇨ This leads to an **asymmetry between the two high tides of the day**.

↪ For instance, consider a maximum declination angle of 28.5°, there will be a large high tide when a point is directly below the bulge. 12.25 hours later, the same point will be on the other side, but the other end of the bulge is down in the opposite hemisphere. As a result, the high tide there will be much smaller than the previous high tide.

▪ At the same latitude (the same position on Earth), there will be **a big high tide** and **a small high tide**. In between the two high tides, the low tides will still be the lowest you can get. This can be observed in the tide gauge record shown in **#WAVES5.1c**.

▪ An **extreme example** of this would be at a slightly higher latitude. The small high tide would be no larger than the low tide. This would result in only one tide per day: a **diurnal signal**.

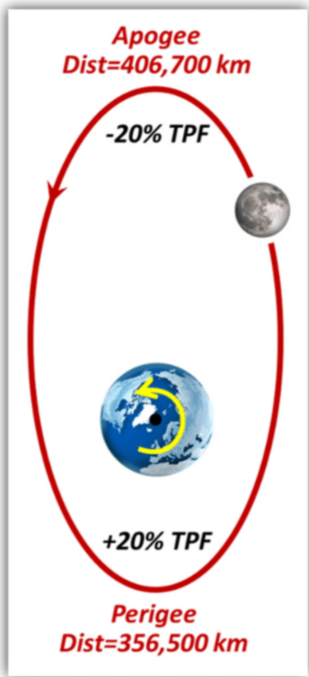
↳ Because of the tilt in the plane of the moon's orbit relative to the equatorial plane, we usually describe the tides as a combination of two signals: **the diurnal tide and the semi-diurnal tide**.

⇨ At maximum declination (28.5°), the diurnal inequality is greatest all over the world, and the tides are known as **tropic tides**. At minimum (zero) declination (the moon is in the equatorial plane), there is no inequality anywhere in the world, and the tides are called **equatorial tides**.

↳ Note that there are other reasons for the diurnal and semi-diurnal components of the tides on Earth (see **#WAVES5.4**). But when the moon is slightly off the equatorial plane, on a spherical Earth completely covered in ocean, diurnal and semi-diurnal tidal forcings already exist.

5.3.c) Moon's elliptical orbit

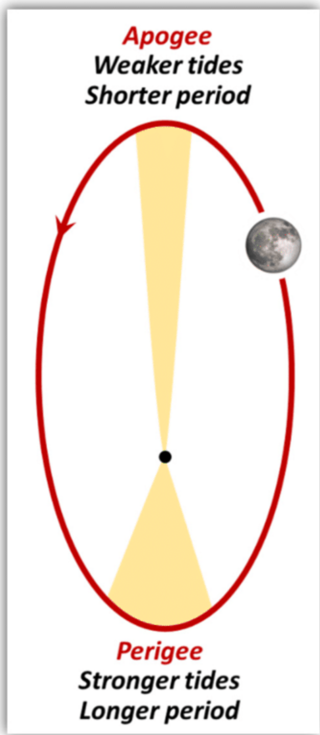
⇨ **Lunar tides** have **semi-diurnal and diurnal components**.



EXCEPT that there is another complication. **The orbit of the moon around the Earth-Moon centre of mass is not circular, but elliptical**, with the earth positioned at one of the foci.

↪ This means that sometimes the moon is closer to Earth and sometimes it is further away. There is a **13% difference** in the Earth-Moon distance between apogee (when the moon is farthest from Earth) and perigee (when the moon is closest to Earth). The closer, the stronger the tides (see **#WAVES5.2b**).

↳ As a result, there is a monthly modulation (of + and – 20%) of the gravitational force exerted by the moon on the earth's surface, and the **amplitude of the tides varies throughout the month**. This can be observed in the tide gauge record shown in **#WAVES5.1c**.



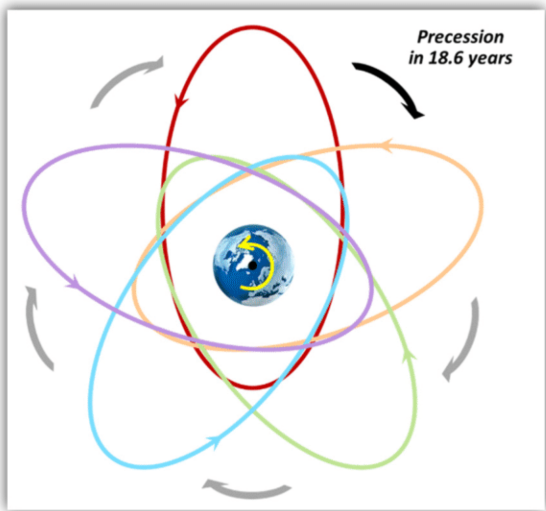
📖 If there is a **full moon** (see **#WAVES5.4b**) when the moon is closest to the Earth (perigee), it is a super moon. This happens at least once a year.

⇨ Because the moon's orbit is elliptical, **Kepler's law** states that **the moon sweeps equal area in equal time**. This means that the moon travels faster when it is close to the earth and it travels slower when it is farther away. As a result, **the tidal period is not fixed to 12 hours and 25 minutes**.

▪ When the moon is closer to the earth, it will move faster and the earth will need more time to catch up with it (see **#WAVES5.3a**). The **lunar tide period will be longer than 12h25'**.

▪ When the moon is farther away, it will move more slowly and the **lunar tide period will be shorter than to 12h25'**.

EXCEPT that this ellipse is not fixed in space. It has a **precession period of 18.6 years**, which introduces a long-timescale into the variations of the tides (period and amplitude) that can be seen in long-term tidal records.



WAVES5.4: Lunar and Solar Tides

5.4.a) Solar tide

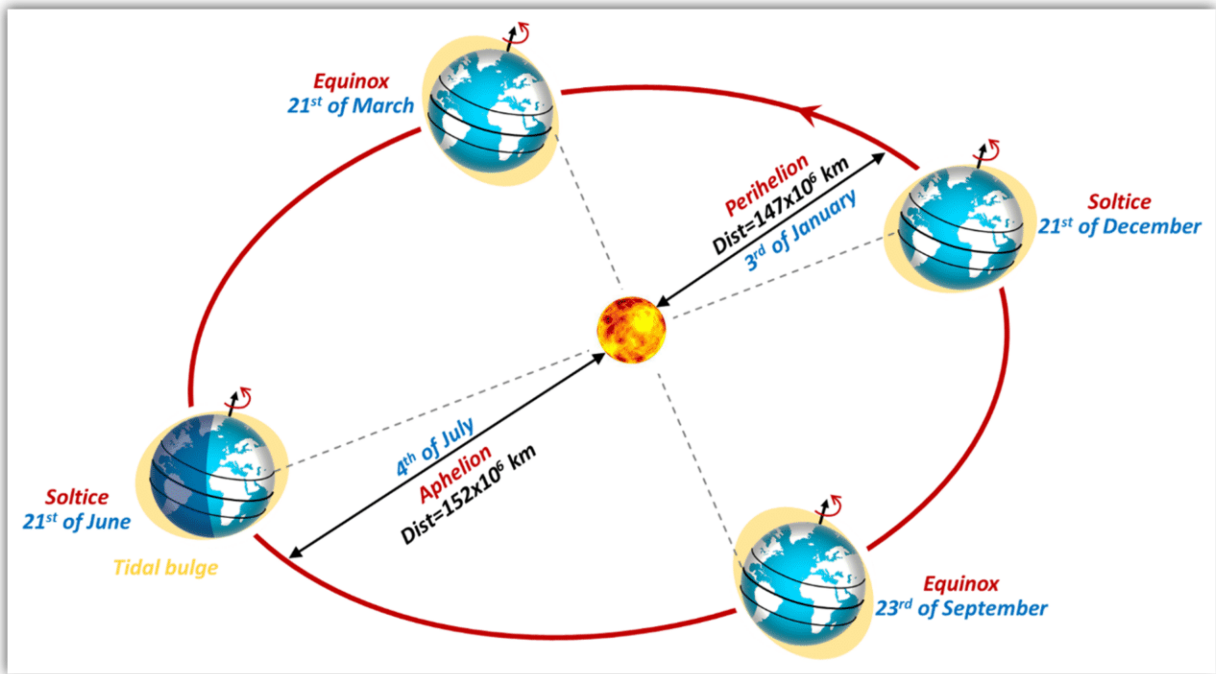
⇨ Let's bring the other player into the picture, **the sun**. Just like the moon, the sun also produces tractive forces. Let's review the characteristics of the solar tides:

• The Tide-Producing Force varies with the mass of the attracting body but is inversely proportional to the cube of the distance from Earth ($TPF \propto M_2/R^3$, see **#WAVES5.2b**).

↪ Although the sun is enormously more massive than the moon ($M_{Sun} = 1.988 \times 10^{30}\ kg$, i.e. 27×10^6 times more mass than the moon), it is 390 times farther from the earth. As a result, the **Tide-Producing Force is about 0.46 that of the moon**.

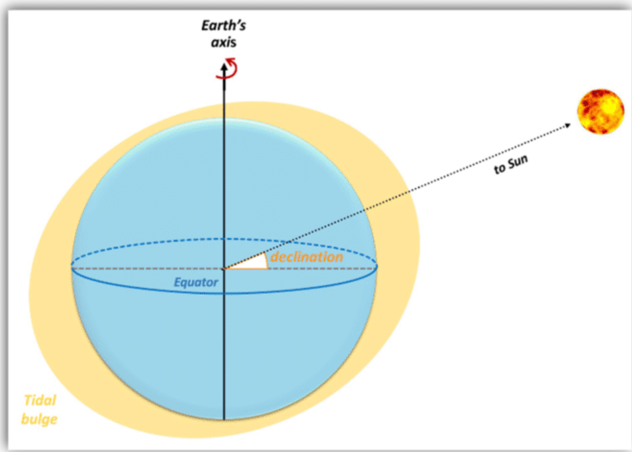
• The gravitational force of the sun also produces an equilibrium tide with two bulges directed towards and away from the sun (as in **#WAVES5.2c**). The solar day is exactly 24 hours, i.e. a sidereal day (23 hours, 56 minutes, 4''), plus ~4 minutes for the earth to catch up with the sun's progression.

↪ The **solar semi-diurnal tide period** is half a solar day, i.e. **exactly 12 hours**.



EXCEPT that:

⇢ Because of the earth's axial tilt in the plane of the ecliptic ($23.4°$), the sun shines at different angles at different latitudes throughout the year, causing the seasons. The angle of declination also leads to an asymmetry between the two high tides of the day and a combination of diurnal and semi-diurnal solar tides (see **#WAVES5.3b**).



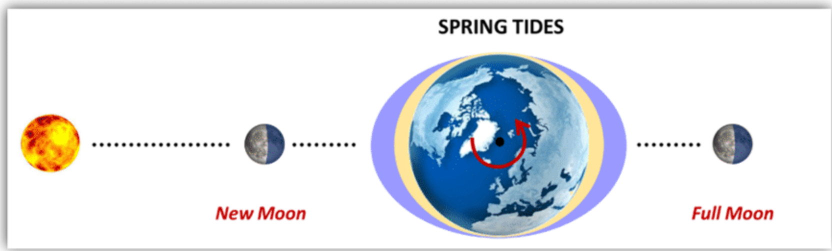
⇢ Because the orbit of the earth around the sun is elliptical, the distance between the earth and the sun varies throughout the year. However, the difference between the smallest (**perihelion**, January 3rd) and the largest (**aphelion**, July 4th) Earth-Sun distance is only $\sim$4%. It causes a slight modulation of the length of a day of $\pm 30''$, which we ignore for timekeeping (this is the Mean in GMT), and a modulation of the tidal magnitude (due to a modulation of the gravitational force) and period (due to a modulation of the speed of the earth: Kepler's effect):

- When the earth is closer to the sun in January, the TPF is slightly larger. The earth is also moving slightly faster and the earth will take more time to catch up with the sun (see **#WAVES5.3a**). The solar tide period will be slightly longer than 12 hours.
- When the earth is further away in July, the TPF is slightly weaker, the earth is slightly slower, and the solar tide period will be slightly shorter than 12 hours.

⇢ Note that the earth's orbit around the sun also changes cyclically on very long timescales ($\sim$10,000 years).

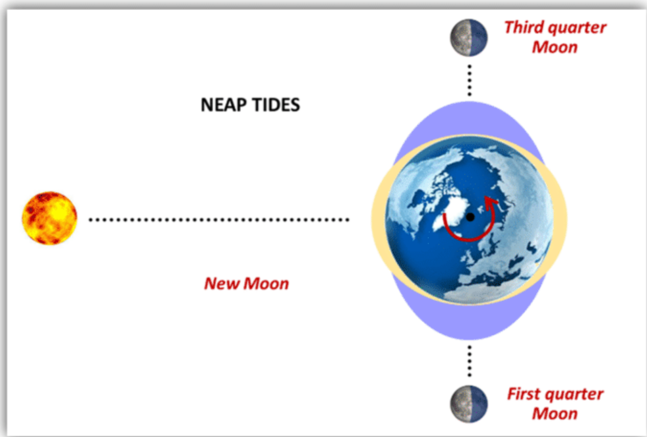
5.4.b) Spring tides and neap tides

⇨ There is interference between the solar and lunar semi-diurnal tides at 12h and 12h25' respectively. This results in a modulation of the signal (as in **#WAVES1.3a**): twice a month there are strong tides and twice a month there are weaker tides. Let's look at some simple cases:



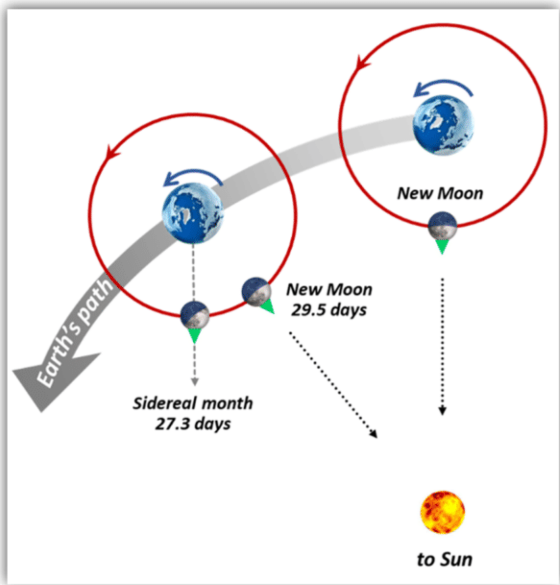
- When the TPF of the sun and the moon act in the same direction (in syzygy), the equilibrium tides coincide. The **tidal bulges are in phase** and reinforce each other. This yields strong tides, where the high tide is higher, and the low tide is lower than the average. Such tides are called **spring tides**.

- A week later, the sun and the moon are at right angles to each other (in quadrature), the **solar and lunar tidal bulges are out of phase** and do not reinforce. The resulting tidal range is smaller than average. It is not zero because the lunar tide is twice as large as the solar tide. These tides are called **neap tides**.



⇘ Twice a month there are **spring tides** and then twice a month there are **neap tides**.

5.4.c) The lunar month



⇨ *What do we mean when we say a month?*

⇢ The earth revolves about the sun in 365.25 days, in the same direction as the moon rotates around the Earth-Moon barycentre. During a **sidereal month** of 27.3 days (the period of the moon's revolution relative to the fixed stars), the earth travels by about 1/13 of its revolution around the sun.

⇢ Starting from a **new moon** configuration (green triangle on the side figure), it would take **an additional** ~**2.2 days** for the moon to catch up with the sun, i.e. to be facing the sun again the next month. This is similar to the earth taking an extra 50 minutes to catch up with the moon's revolution during a sidereal day (see **#WAVES5.3a**).

↳ The **synodic month**, often called the **lunar month** is the period between two successive new moons. It counts **29.5 days**.

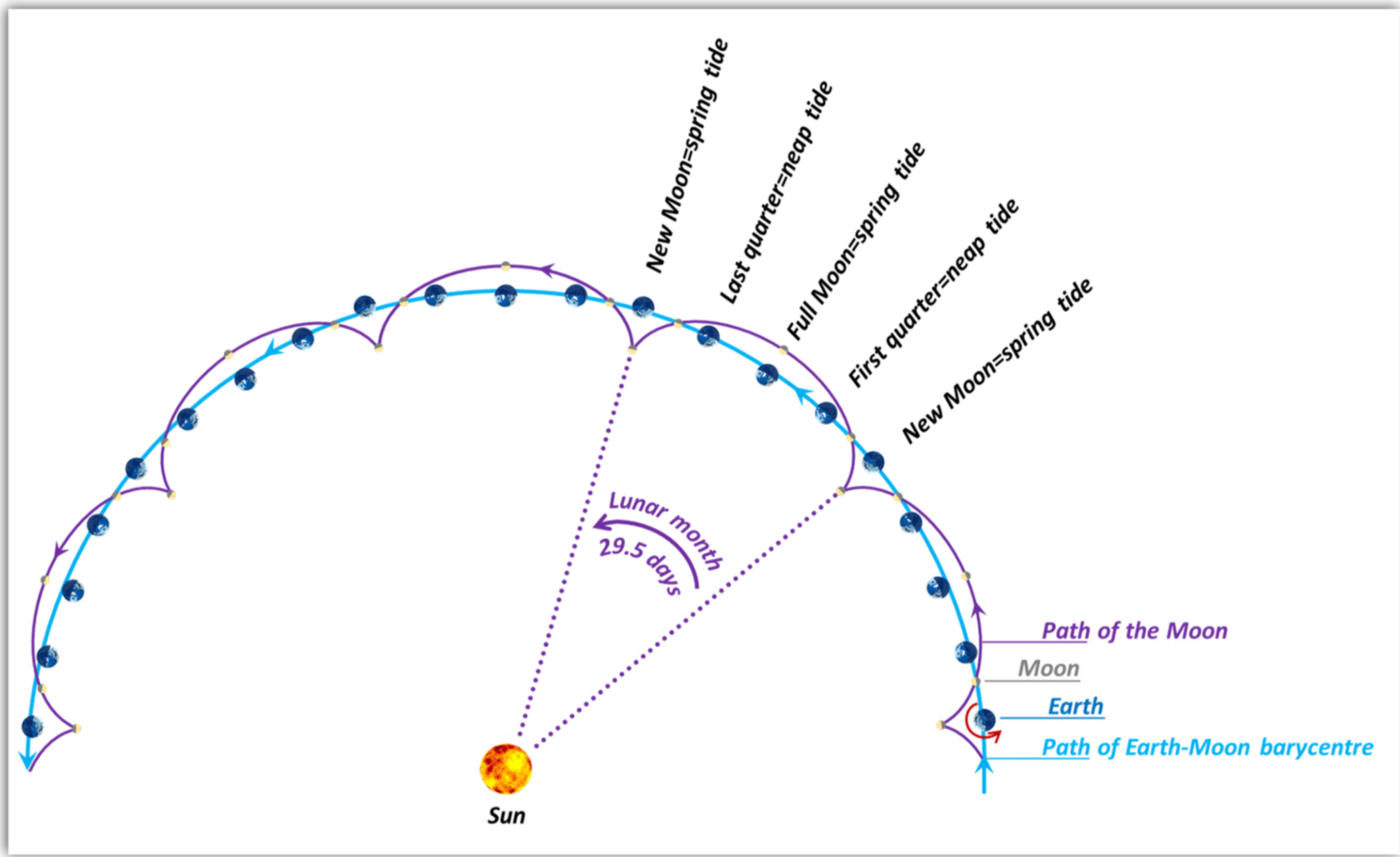
⇨ The diagram below is a summary of the combined motions of the earth and the moon about the sun.

▪ It shows the revolution of the moon around the Earth-Moon centre of mass as both bodies orbit the sun. The moon traces out a **trochoid** on an ellipse.

↪ It resembles the trajectory of water particles in a surface wave (see **#WAVES2.1e**).

▪ The earth's mass centre wobbles a little bit because the earth is also dancing around the barycentre of the Earth-Moon system.

▪ The diagram also illustrates the lunar month (29.5 days) spring-neap tidal cycle.



WAVES5.5: Tides in Ocean Basins

⇨ In this section, we will move away from the concept of a spherical earth covered in ocean and look at **real oceans**. The patterns associated with tides are quite complex because:

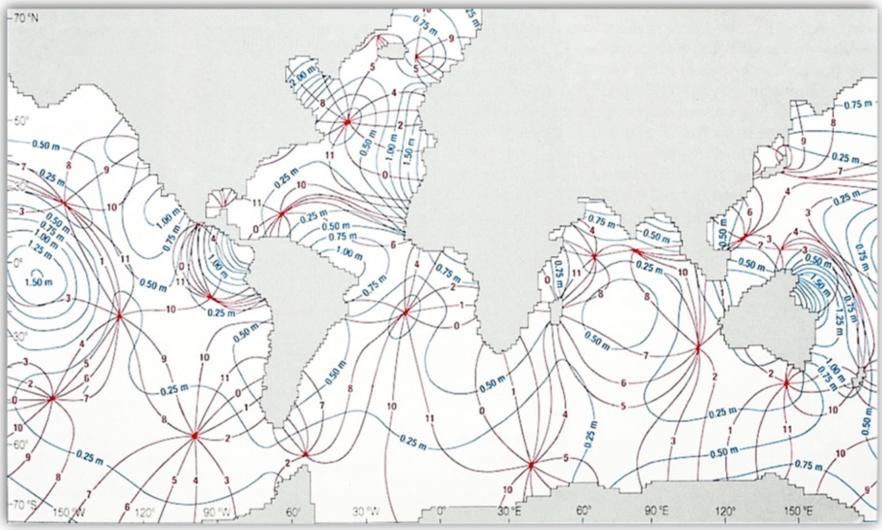
▪ The **presence of the continents** prevents the tidal bulge from directly circumnavigating the globe (except in the Southern Ocean, around Antarctica).

▪ The earth rotates too fast on its axis for the shallow-water wave to catch up with the moon or the sun's gravitational forces at semi-diurnal frequencies (except at latitudes higher than 65°). As a result, there is a **time lag in the response of the oceans** to the semi-diurnal tractive force which is greatest at the equator.

▪ The movement of water is subject to the **Coriolis force**, which deflects the currents to the right in the Northern Hemisphere and to the left in the Southern Hemisphere.

5.5.a) Amphidromic system: M_2 tide

⇨ Below is a world map of the propagation pattern associated with the dominant semi-diurnal lunar tide. This map shows sea level variations with a frequency of 12 hours and 25 minutes. The blue lines show the amplitude of the tidal wave and the red lines show the phase (in lunar hours).



• **The maximum amplitudes are near the coasts**. They **propagate** with a period of 12h25', so that the maximum tide is observed in different places at different times of the day.

↪ In the eastern Atlantic, the lunar semi-diurnal tide will reach its maximum at phase 0 south of Spain and at its maximum two hours later north of France. The wave will propagate anti-clockwise in the North Atlantic basin. The tide will be peak south of Iceland 3 hours later and off *Newfoundland*, Canada at phase 9.

• There are regions where the amplitude of the tidal wave is zero and the phase lines meet. These points are called **amphidromic nodes**. The tidal range is zero at each amphidromic node and increases outwards, usually to a maximum at the coast. The crests and troughs of the tidal wave circulate around a nodal point during each tidal period, forming an **amphidromic system**.

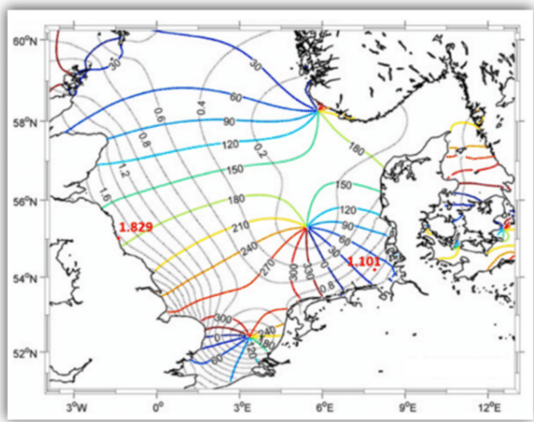
⇢ There is an amphidromic node in the North Atlantic basin, around which the semi-diurnal lunar tidal wave propagates anti-clockwise.

⇢ There is an amphidromic node in the southeastern Pacific Ocean (at ~[100°W; 58°S]) and one near Antarctica (~ at [40°E, 65°S]) around which the semi-diurnal tidal wave rotates clockwise.

↪ The **direction of propagation** of the tidal wave around the amphidromic node depends on the actual **shape of the coasts** and the arrangement of the ocean basins, but a simple explanation would be that the tidal wave propagates anti-clockwise in the northern hemisphere and, it propagates clockwise in the southern hemisphere, with some exceptions.

➢➢ *Why would the phase propagate anti-clockwise around a nodal point in the northern hemisphere?*

5.5.b) Coriolis deflection



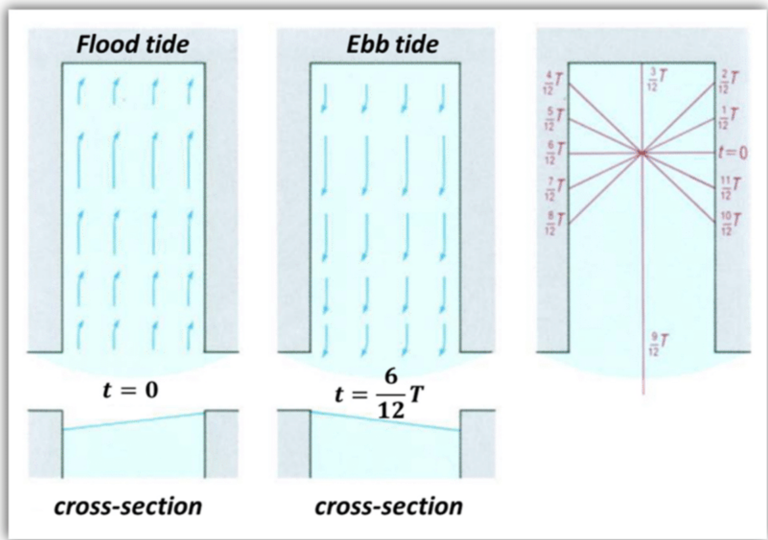
⇨ Tidal waves are shallow-water gravity waves on a rotating planet, forced by the gravitational forces of the moon and sun. They propagate along the coast as **boundary Kelvin waves** (see **#WAVES3.3**). The direction of propagation is controlled by the Coriolis force.

↳ In the Northern Hemisphere, boundary Kelvin waves are trapped at the coast, with the coast to their right as they propagate (see **#WAVES3.3b**). In the Southern Hemisphere, they propagate clockwise.

⇨ Here is another way to show why a Kelvin wave propagates this way.

⇢ Imagine water moving into a closed basin in response to the moon's gravitational pull (flood tide). The tidal current experiences the Coriolis force and gets deflected to the right (in the northern hemisphere). The water piles up on the right-hand coast.

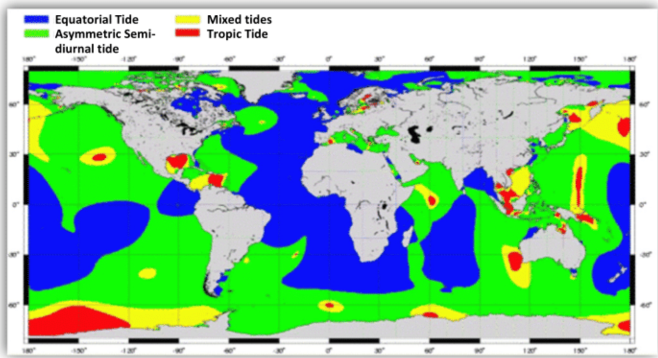
⇢ 6 lunar hours later, the water is pulled out by the tidal attraction (ebb tide), and the current is again deflected to its right by the Coriolis force. The water piles up against the opposite wall.



↳ In the course of a tidal cycle, because the water is constrained by the land masses, an anti-clockwise amphidromic system develops. This is a Kelvin wave.

5.5.c) Mixed tides

⇨ Combined equatorial and tropic tides and lunar and solar tides give very different tidal signatures at different locations. There are regions where the sea level variations are dominated by the semidiurnal tides, regions where the diurnal signal prevails, and regions where there is a mixed signal.



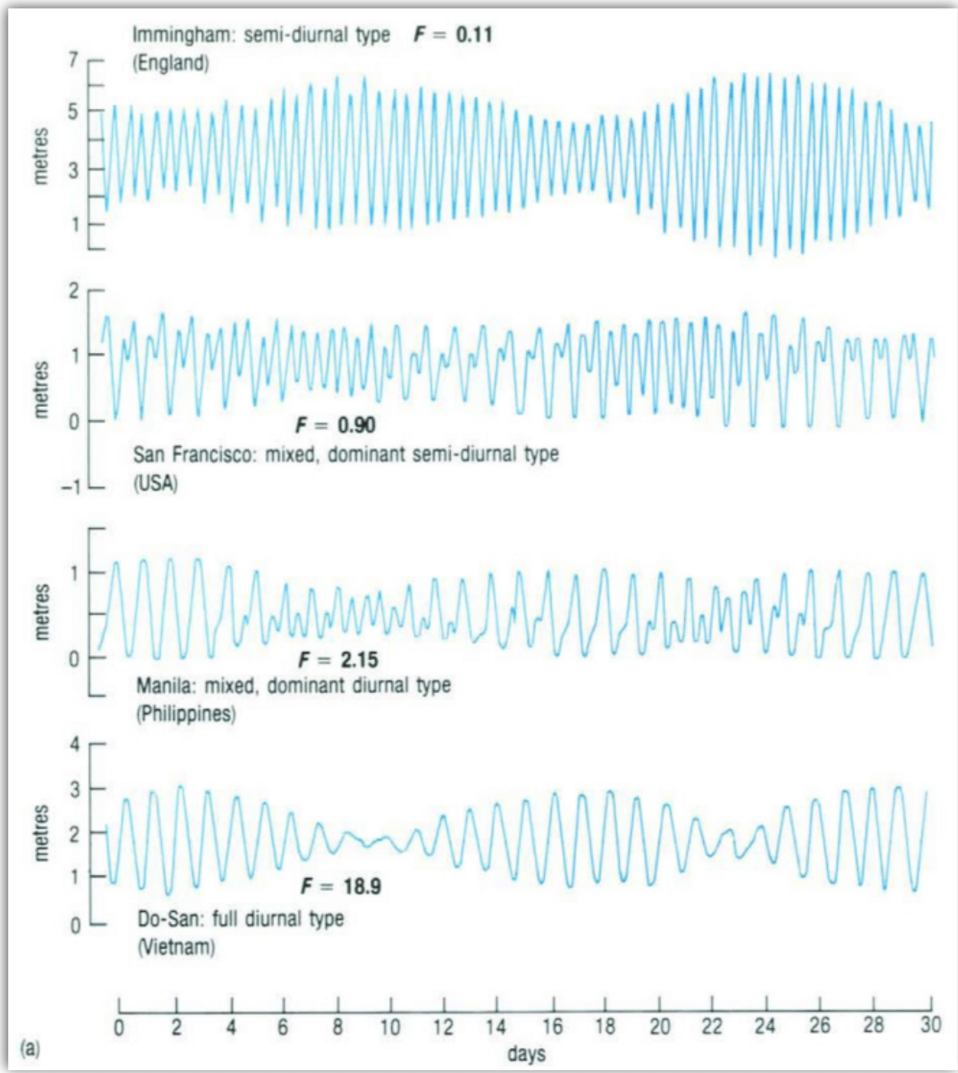
⇨ Here are some examples of 30-day tidal records measured at different locations around the world. They highlight the **variety** of the possible combinations (amplitude and phase) of diurnal and semi-diurnal frequencies.

⇢ At *Immingham* (England), there is a very clear semi-diurnal component (high and low tides occurring twice a day) and a 15-day modulation (similar to the tidal record shown in **#WAVES5.1c**).

⇢ In *San Francisco* (California, USA), the tidal record is mostly semi-diurnal (days 18-24), but sometimes the diurnal tide emerges (days 1-18). The two frequencies are mixed, but on average semi-diurnal component dominates.

⇢ In *Manila* (Philippines), the mixture of the diurnal and semi-diurnal components is dominated by the diurnal type.

⇢ In *Do-San* (Vietnam), there is a very clear diurnal component to the tides. Not necessarily because of latitude (as discussed in **#WAVES5.3b**), but because of the size of the ocean basin and the time it takes for a boundary Kelvin wave to travel around it (resonance period).



5.5.d) Fourrier coefficients

⇨ To analyse or to predict tides, it is necessary to record the sea level at specific locations and decompose the time series into partial tidal components, i.e., disentangle the frequency constituents associated with each component of the tides, e.g. solar and lunar semi-diurnal and diurnal components. As many as 390 frequencies have been identified and included in tidal models.

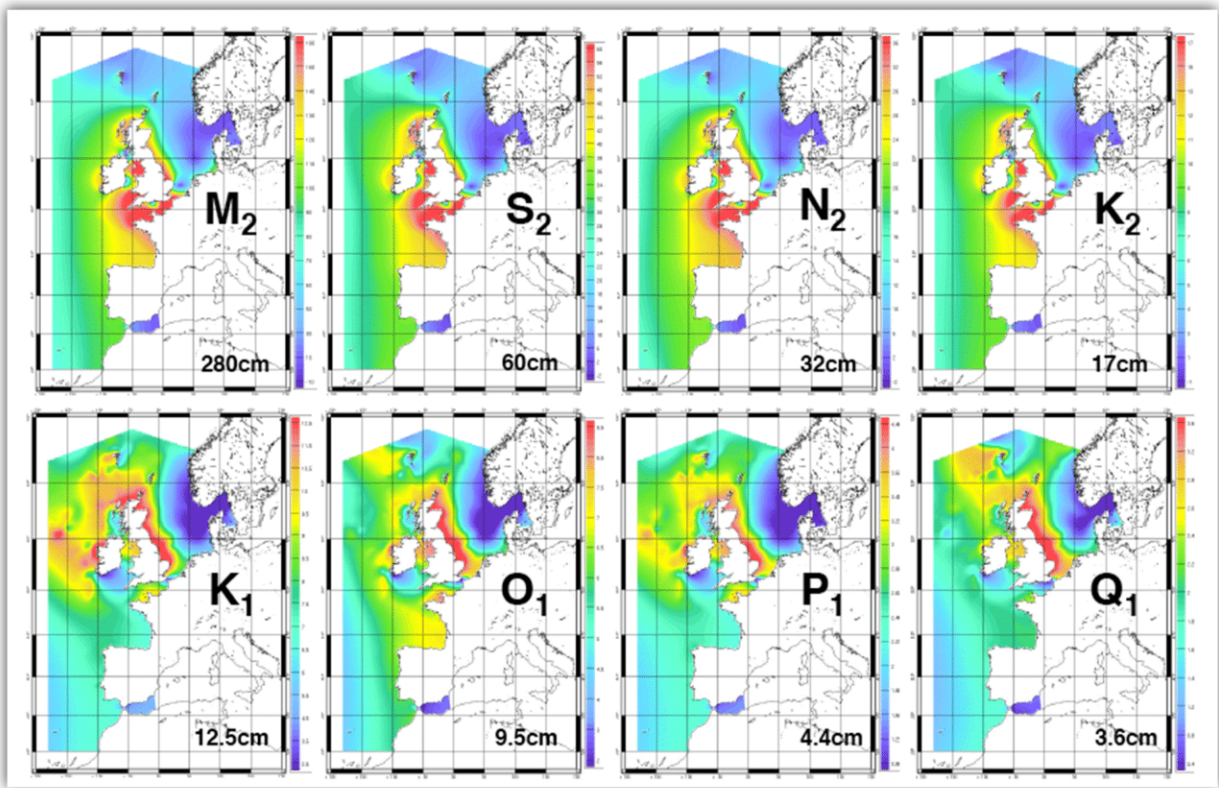
↪ Each harmonic has been given a name. For example, the solar and lunar semi-diurnal tides at periods 12h and 12h25' are called S_2 and M_2 respectively.

↪ The 9 most important tidal components are listed in the following table for semi-diurnal, diurnal, and shorter frequencies. Their amplitude is given with respect to the M_2 tide.

Tidal components	Symbol	Periods (hours)	Coefficients ratio
Semi-diurnal :			
Principal Lunar	M_2	12.42	100
Principal Solar	S_2	12.00	46.6
Larger Lunar elliptic	N_2	12.66	19.2
Luni-solar	K_2	11.97	12.7
Diurnal :			
Luni-solar	K_1	23.93	58.4
Principal lunar	0_1	25.82	41.5
Principal solar	P_1	24.07	19.4
Longer Periods:			
Lunar fortnightly	M_f	327.86	17.2
Lunar monthly	M_m	661.30	9.1

5.5.e) Tides around Europe

⇨ Below are maps of **the tidal amplitude around Europe** in the North Atlantic for 4 semi-diurnal components (M_2, S_2, N_2, and K_2, upper panels) and 4 diurnal components (K_1, 0_1, P_1, and Q_1, lower panels) of the tides.



⇢ Not surprisingly, the **patterns** of the semi-diurnal components of the tides all **look quite similar**. This is because the maps represent the **response of a dynamical system** to a forcing at a similar frequency. The system does not care which astronomical object is doing the forcing, it responds in the same way.

⇢ Conversely, the **tidal range** of each semi-diurnal component differs because each astronomical object exerts a different Tide-Producing Force on the surface of the earth, the amplitude of which depends on the object's mass and is inversely proportional to the cube of the distance to the object (see **#WAVES5.2b**).

↪ The patterns of the diurnal component are quite different from the semi-diurnal ones, but again all these diurnal patterns look similar.

5.5.f) Tidal range

⇨ Let's go back, to the tidal range in the English Channel (see **#WAVES3.3b**).

↳ Because the tide propagates as a shallow-water coastal-trapped Kelvin wave, its amplitude is only a few meters on the English side, but it exceeds 11 meters on the French coast, near *Saint-Malo*. A tidal power station was built in the *Rance* estuary in 1963 to take advantage of the large daily tidal variations.