















# Parameterization

Sophisticated parameterization schemes use physical assumptions to determine the elements of the tensor K.

This often involves a symmetric part and an antisymmetric part so the scheme will be equivalent to a diffusion along the gradient of the tracer plus an advection by a residual flow.

An example is the **Gent and McWilliams** scheme, which hypothesizes energy conversion by baroclinic instability and formulates the eddy closure in terms of asymmetric diffusion of thickness. Such a scheme will tilt density contours to liberate available potential energy, rather than just erode gradients.

Other schemes have been formulated in terms of potential vorticity diffusion, or in terms of flow dependent coefficients of K.



Whether or not it is appropriate to use a downgradient diffusion depends on the quantity being diffused. We might expect conserved quantities to behave like tracers, with diffusion eroding their gradients. For non-conserved quantities more elaborate schemes need to be considered.





# Stommel vs Fofonoff

Two extreme paradigms of gyre-scale flow:

A Stommel gyre has different vorticity balance in different regions

$$eta V = rac{1}{
ho} 
abla_\wedge au_s - R 
abla_\wedge V$$

Flow is forced south across contours of planetary vorticity. Its changing vorticity is supplied by forcing and dissipation.

A Fofonoff gyre corresponds to unforced flow, so the balance is between advection of planetary vorticity and relative vorticity.

$$v.
abla \xi + eta v = v.
abla q = 0$$
 or  $J(\psi,q) = 0, \ q = q(\psi)$ 

so 
$$\nabla^2 \psi + \beta y = fn(\psi)$$

Flow conserves its absolute vorticity, so contours of absolute vorticity are parallel to contours of the streamfunction.





# Diffusion and the strength of the gyre

In a Fofonoff gyre we don't know the relationship between q and  $\psi$ , so the strength of the flow is not constrained. Let's assume that the relation is linear, and reintroduce some forcing and downgradient diffusion of q.

$$abla q pprox rac{dq}{d\psi} 
abla \psi, \quad J(\psi, q) = 
abla.(\kappa 
abla q) + S$$

Integrating within a streamline of  $\psi$ :  $0 = \iint_A \nabla . (\kappa \nabla q) \, dA + \iint_A \mathcal{S} \, dA$ 

$$\Rightarrow \quad \iint_A \mathcal{S} \, dA = -\oint_{\psi} \kappa \nabla q. \hat{\mathbf{n}} \, dl = -\oint_{\psi} \kappa \frac{dq}{d\psi} \nabla \psi. \hat{\mathbf{n}} \, dl$$

so 
$$\frac{dq}{d\psi} = -\frac{\int\!\!\!\int_A \mathcal{S} \, dA}{\oint_\psi \kappa \, \mathbf{v}. dl}$$

The relationship between q and ψ is determined by integrals of forcing and dissipation around the closed gyre circulation.
Integrated eddy diffusion provides the link between the q / ψ relationship and the strength of the circulation.
In regions isolated from forcing, the numerator is zero but the denominator is non-zero, so the field of q must be uniform. q is homogenized.







(Overland et al 2008)

### The importance of nonlinearity Monthly mean 500mb height There is no doubt that atmospheric dynamics is nonlinear. One need only look at the difference between cyclones and anticyclones. Does this mean we need a nonlinear framework to analyse lower frequency variability ? Non-Gaussian and even multi-modal statistics are features of nonlinear systems. But synoptic timescale nonlinearity can be represented as EOF2 stochastic noise plus linear damping. The response of a linear system to external forcing can be written: EOF1 $\frac{dx}{dt} = \underbrace{Lx}_{\text{state vector}} + \underbrace{f}_{\text{external forcing}}$ Gaussian noise Lorenz system This linear system yields Gaussian statistics if B is constant, but can deliver non-Gaussian unimodal statistics if B=B(x).

(Sardeshmukh and Sura, 2009)







# Scale separation and boundary conditions