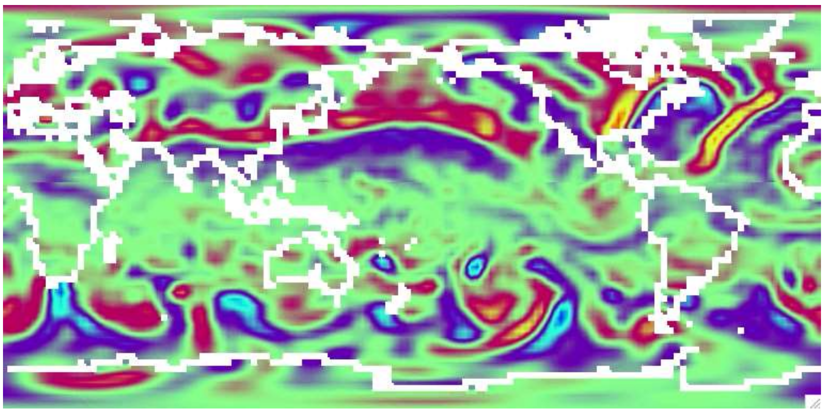
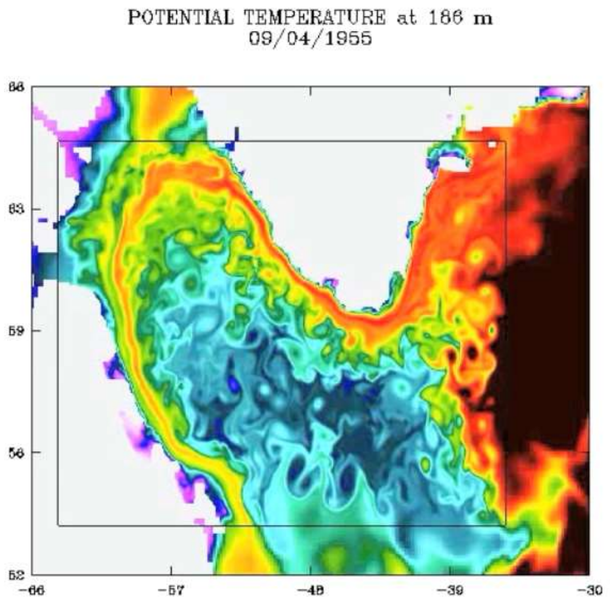
Scale interactions in the atmosphere and ocean

- ⇨ Large-scale forcing and transport due to transient systems
- ⇨ Closure and diffusion
- ⇨ Modification of large-scale potential vorticity: Impact on ocean circulation.
- ⇨ Long-lived features of the atmosphere and low frequency variability.
- ⇨ Atmospheric response to forcing anomalies.
- ⇨ The wave-turbulence crossover and zonal jets

250 mb relative vorticity (ERAi DJF)



Restratification of the Labrador Sea (MEOM)



(Chanut et al 2008)

General considerations for tracer transport

⇨ So far we have looked at small perturbations, linear systems, oscillating or exponentially growing / decaying solutions.

↳ Consider nonlinear system $\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{F} - \mathcal{D}$ or $\frac{\partial q}{\partial t} + J(\psi, q) = \mathcal{F} - \mathcal{D}$

$$u = -\frac{\partial \psi}{\partial y} \qquad v = \frac{\partial \psi}{\partial x}$$

⇨ Tendency equation for q $q = \overline{q} + q' \qquad \psi = \overline{\psi} + \psi'$

$$\frac{\partial q'}{\partial t} + \underbrace{J(\overline{\psi}, \overline{q})}_{\text{mean flow advection}} + \underbrace{J(\overline{\psi}, q') + J(\psi', \overline{q})}_{\text{linear waves}} + \underbrace{J(\psi', q')}_{\text{turbulence}} = \mathcal{F} - \mathcal{D}$$

⇨ Budget equation for $\overline{q}$ $J(\overline{\psi}, \overline{q}) = -\overline{J(\psi', q')} + \overline{\mathcal{F}} - \overline{\mathcal{D}}$

transient "forcing"

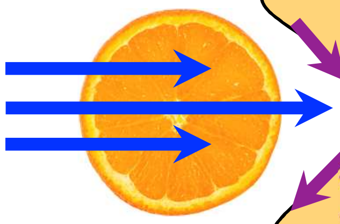
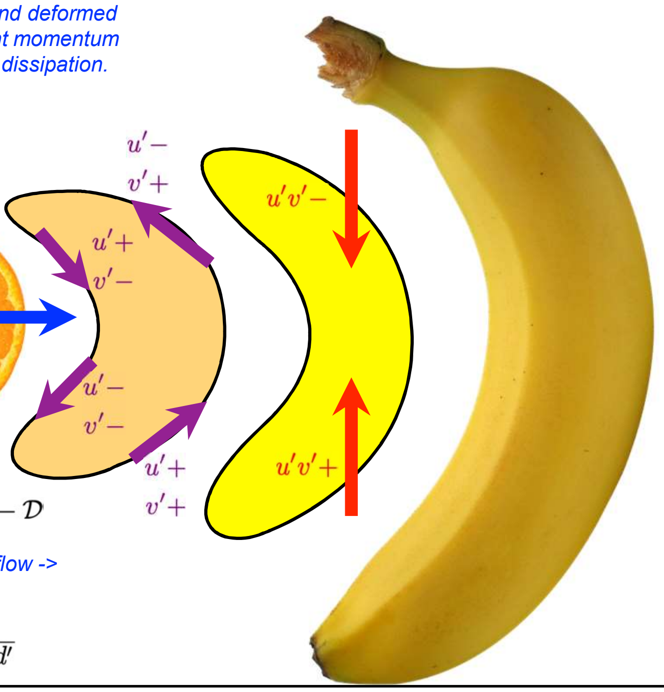
⇨ If we have steady unforced flow $J(\psi, q) = 0 \quad \Rightarrow \quad q = q(\psi)$

↳ This describes a closed circulation - q contours coincide with ψ contours.
So nonlinearity is associated with closed circulations

If the closed circulation is large scale, this describes a gyre.
If the closed circulation is small scale, we need to relate it to the large-scale circulation.

Example: Momentum transport in zonal jets

An eddy that gets stretched out and deformed by a jet, will produce a convergent momentum flux that maintains the jet against dissipation.

$$u_t + uu_x + vu_y = f(v - v_g) - \mathcal{D}$$

Geostrophic nondivergent mean flow ->

$$\overline{u'u'_x} + \overline{v'u'_y} = -\overline{\mathcal{D}}$$

$$(\overline{u'u'})_x + (\overline{u'v'})_y = -\overline{\mathcal{D}} + \overline{u'd'}$$

Forcing due to transients: Closure

⇨ *Imagine we wish to simulate or predict the slow,* ***large-scale flow****. Because the system is nonlinear the fast, small-scale component (maybe unresolved) will affect the slow, large scale.*

↳ *Consider the nonlinear system* $\frac{du}{dt} + uu + ru = 0$

⇨ *The average is* $\frac{d\overline{u}}{dt} + \overline{uu} + r\overline{u} = 0$

↳ *But the problem is that* $\overline{uu} \neq \overline{u}\,\overline{u}$ *it's* $\overline{uu} = \overline{u}\,\overline{u} + \overline{u'u'}$

⇨ *Multiply the equation by u and take time mean* $\frac{1}{2}\frac{d}{dt}\overline{uu} + \overline{uuu} + r\overline{uu} = 0$

↳ *This gives us an equation for* $\overline{uu}$*, but now we have a cubic term!* ☹

In general we need to represent the (n + 1)th order term in terms of the nth order term.
We must make additional physical assumptions to do this.

⇨ *What is the relation between the transport of transients and the mean flow?*

Diffusion and diffusivity

⇨ Let's go back to our tracer equation and consider a diffusive representation for the flux of the tracer q. For the moment we ignore other forms of forcing and dissipation. Consider advection by a **nondivergent flow**:

$$\frac{\partial q}{\partial t} + \nabla.\mathbf{v}q = 0$$ take an ensemble average $$\frac{\partial \overline{q}}{\partial t} + \nabla.\overline{\mathbf{v}}\,\overline{q} = -\nabla.\overline{\mathbf{v}'q'}$$

⇨ Let's represent this eddy covariance through **analogy with molecular diffusion**, i.e. transport down the mean gradient:

$$\overline{\mathbf{v}'q'} = -K\nabla\overline{q}$$

↳ So $\frac{D\overline{q}}{Dt} = \nabla.(K\nabla\overline{q}) \quad (= \nabla.F)$ where $\boldsymbol{F}$ is the diffusive flux of q.

⇨ In general, K is a second rank tensor. It is usually **not isotropic** for large-scale flows.

↳ For example $\overline{\mathbf{v}'q'} = -\kappa^{vy}\frac{\partial \overline{q}}{\partial y} - \kappa^{vz}\frac{\partial \overline{q}}{\partial z}$

↳ We can estimate $\kappa^{vy} \sim v'l'$ where v' is a typical eddy velocity and l' is a"mixing length" (various scalings can be used).

But it can be more complicated…

Symmetric and asymmetric diffusion

⇨ If we decompose K into symmetric and antisymmetric parts $\boldsymbol{K} = \boldsymbol{S} + \boldsymbol{A}$.

⇨ **Isotropic diffusion corresponds to**

$$K = S = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$ and the diffusive flux $F = -\kappa\nabla q$ is downgradient.

⇨ But **if K is antisymmetric** $F = -A\nabla q$ and the flux is **parallel to contours of q**.

(if A has zero diagonal and opposite sign off-diagonal elements then $A\boldsymbol{x} \perp \boldsymbol{x}$)

$$F.\nabla q = -(A\nabla q).\nabla q = 0$$

↳ The flux is neither upgradient nor downgradient. It's called a "**skew flux**".

↳ A skew flux is equivalent to **advection** by a nondivergent flow with velocity $\tilde{\mathbf{v}} = \nabla_{\wedge}\psi$
The elements of A can be expressed in terms of ψ

⇨ Whether or not it is appropriate to use a downgradient diffusion depends on the quantity being diffused.
We might expect conserved quantities to behave like tracers, with diffusion eroding their gradients.
For non-conserved quantities more elaborate schemes need to be considered.

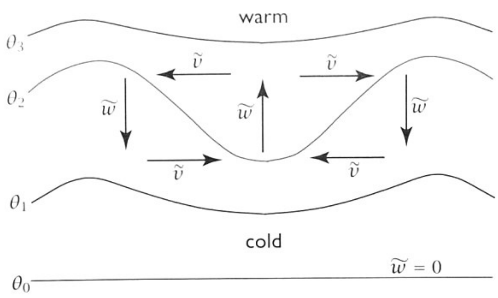
Parameterization

Sophisticated parameterization schemes use physical assumptions to determine the elements of the tensor K.

This often involves a symmetric part and an antisymmetric part so the scheme will be equivalent to a diffusion along the gradient of the tracer plus an advection by a residual flow.

An example is the ***Gent and McWilliams scheme****, which hypothesizes energy conversion by baroclinic instability and formulates the eddy closure in terms of asymmetric diffusion of thickness. Such a scheme will tilt density contours to liberate available potential energy, rather than just erode gradients.*

Other schemes have been formulated in terms of potential vorticity diffusion, or in terms of flow dependent coefficients of K.



⇨ *Whether or not it is appropriate to use a downgradient diffusion depends on the quantity being diffused. We might expect conserved quantities to behave like tracers, with diffusion eroding their gradients. For non-conserved quantities more elaborate schemes need to be considered.*

Potential vorticity homogenization

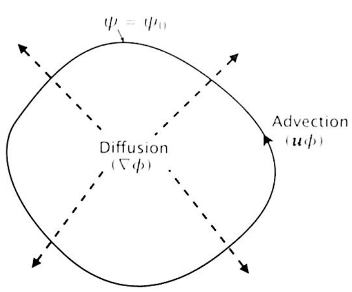
⇨ *Let's look again at our tracer equation for q, and add in some downgradient diffusion*

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \nabla.(\kappa \nabla q) + \mathcal{S}$$

⇨ *For* ***steady nondivergent*** *flow in a region* ***isolated*** *from the source S*

$$\nabla.(\mathbf{v}q) = \nabla.(\kappa\nabla q)$$

⇨ ***Integrate*** *over a region enclosed by a contour of q.*

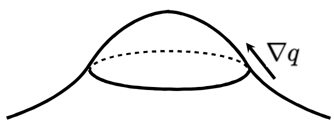


- *The left hand side integrates to zero*

$$\iint_A \nabla.(\mathbf{v}q)\,dA = \oint(\mathbf{v}q).\hat{\mathbf{n}}\,dl = q\oint \mathbf{v}.\hat{\mathbf{n}}\,dl = q\iint_A \nabla.\mathbf{v}\,dA = 0$$

- *So the right hand side must also be zero*

$$\iint_A \nabla.(\kappa\nabla q)\,dA = \oint \kappa\nabla q.\hat{\mathbf{n}}\,dl = 0$$



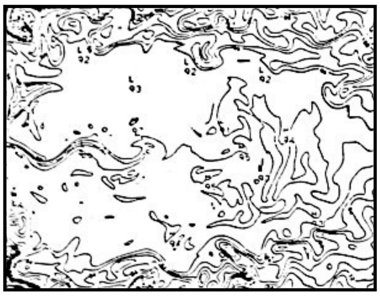
Away from forcing (in depth)

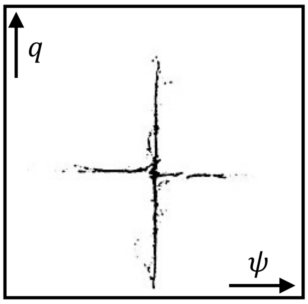
⇨ ***This cannot be true if the contour encloses an extremum of q.***
The gradient of q must integrate to zero around this contour.
So there can be no extremum of q within the contour.

⇨ *Gradients of q are eliminated, resulting in* ***Homogenization*** *to a uniform value in regions remote from sources of q.*

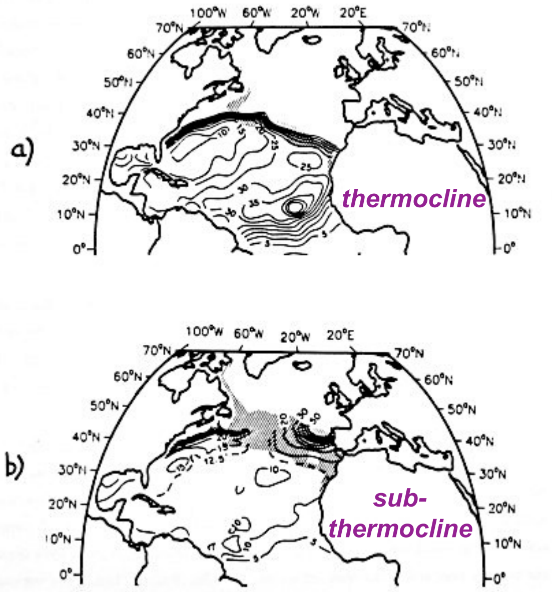
Examples in models and observations

QG model - mid-level PV





Observed PV on isopycnal surfaces



(Keffer 1985)

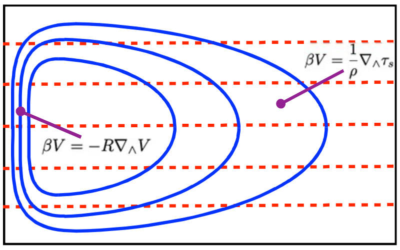
Stommel vs Fofonoff

Two extreme paradigms of gyre-scale flow:

A Stommel gyre has different vorticity balance in different regions

$$\beta V = \frac{1}{\rho}\nabla_\wedge \tau_s - R\nabla_\wedge V$$

Flow is forced south across contours of planetary vorticity. Its changing vorticity is supplied by forcing and dissipation.

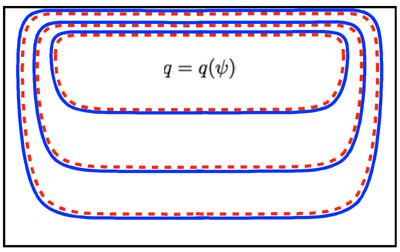


A Fofonoff gyre corresponds to unforced flow, so the balance is between advection of planetary vorticity and relative vorticity.

$$v.\nabla\xi + \beta v = v.\nabla q = 0 \quad \text{or} \quad J(\psi, q) = 0, \quad q = q(\psi)$$

so $\nabla^2\psi + \beta y = fn(\psi)$

Flow conserves its absolute vorticity, so contours of absolute vorticity are parallel to contours of the streamfunction.



Diffusion and the strength of the gyre

In a Fofonoff gyre we don't know the relathionship between q and ψ, so the strength of the flow is not constrained. Let's assume that the relation is linear, and reintroduce some forcing and downgradient diffusion of q.

$$\nabla q \approx \frac{dq}{d\psi}\nabla\psi, \quad J(\psi, q) = \nabla.(\kappa\nabla q) + \mathcal{S}$$

Integrating within a streamline of ψ: $$0 = \iint_A \nabla.(\kappa\nabla q)\, dA + \iint_A \mathcal{S}\, dA$$

$$\Rightarrow \quad \iint_A \mathcal{S}\, dA = -\oint_\psi \kappa\nabla q.\hat{\mathbf{n}}\, dl = -\oint_\psi \kappa\frac{dq}{d\psi}\nabla\psi.\hat{\mathbf{n}}\, dl$$

so $$\frac{dq}{d\psi} = -\frac{\iint_A \mathcal{S}\, dA}{\oint_\psi \kappa\, \mathbf{v}.dl}$$

- The relationship between q and ψ is determined by integrals of forcing and dissipation around the closed gyre circulation.
- Integrated eddy diffusion provides the link between the q / ψ relationship and the strength of the circulation.
- In regions isolated from forcing, the numerator is zero but the denominator is non-zero, so the field of q must be uniform. q is homogenized.

Long-lived atmospheric flow anomalies

Can we imagine similar mechanisms at work within closed atmospheric circulations ?

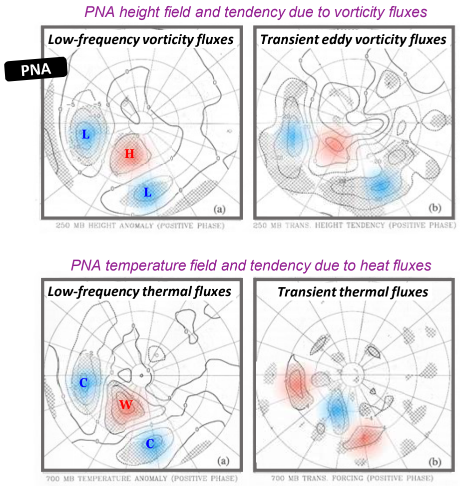
How are low-frequency patterns in the atmosphere maintained against dissipation ?

Transient fluxes of heat and vorticity have rotational and divergent components.

The divergent components are associated with development or maintenance of long-lived structures.

Observational analyses consistently show that high-frequency transient eddy vorticity fluxes reinforce the low-frequency patterns, while transient thermal fluxes dissipate them.

Some anomaly structures may be well configured for maintenance by transients



(Sheng et al 1998)

Transient potential vorticity flux divergence in an idealised model of atmospheric blocking

(Haynes and Marshall 1986)

Transient feedback on a forced response

Imagine the generic development

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{F} - \mathcal{D}$$

Time average of this

$$\overline{\mathbf{v}.\nabla q} = \overline{\mathcal{F}} - \overline{\mathcal{D}} = \mathcal{G}$$

"Forcing" for mean flow can be written

$$\overline{\mathbf{v}}.\nabla\overline{q} = \overline{\mathcal{F}} - \overline{\mathcal{D}} - \overline{\mathbf{v}'.\nabla q'} = \mathcal{H}$$

Use G to force an empirical GCM

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{G}$$

Add a perturbation to the forcing (say an SSTA)

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{G} + f'$$

SET1

The difference between runs gives the average response Δq.

We can also diagnose the difference in transient forcing Δ(v'.∇q').

Now run the same model but force with H, and initialise with $\overline{q}$

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{H}$$

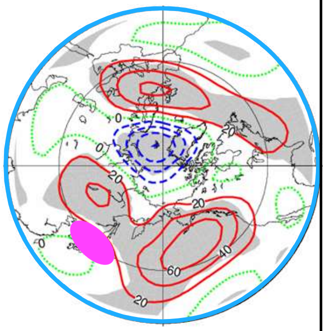
No development !

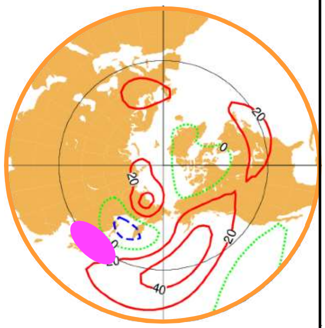
Now add the perturbation.

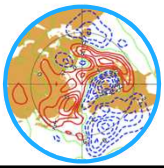
$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{H} + f'$$

SET2

The response is not the same as before.







(Hall et al 2001)

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$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{H}$$

No development !

Now add the perturbation.

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{H} + f'$$

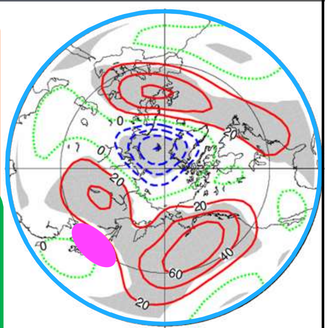
SET2

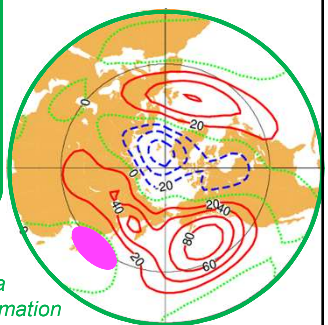
The response is not the same as before.

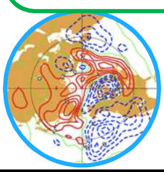
But if we add the extra transient forcing

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{H} + f' - \Delta(\overline{v'.\nabla q'})$$

SET3







The linear model gives a good approximation to the full response

(Hall et al 2001)

The importance of nonlinearity

There is no doubt that atmospheric dynamics is nonlinear. One need only look at the difference between cyclones and anticyclones.

Does this mean we need a nonlinear framework to analyse lower frequency variability ?

Non-Gaussian and even multi-modal statistics are features of nonlinear systems.

But synoptic timescale nonlinearity can be represented as stochastic noise plus linear damping.

The response of a linear system to external forcing can be written:

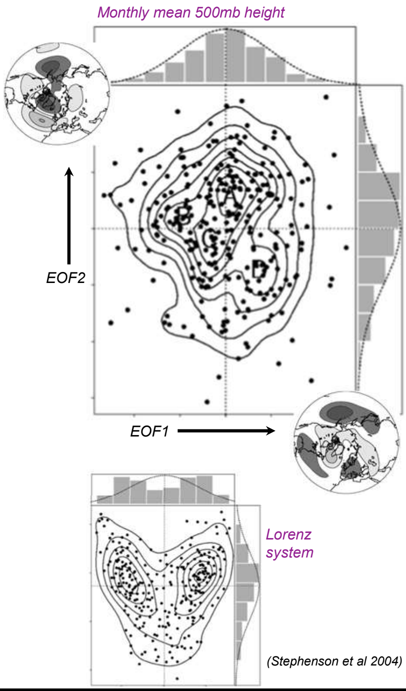
linear operators

$$\frac{dx}{dt} = Lx + f + B\eta$$

state vector *external forcing* *Gaussian noise*

This linear system yields Gaussian statistics if B is constant, but can deliver non-Gaussian unimodal statistics if B=B(x).

(Sardeshmukh and Sura, 2009)

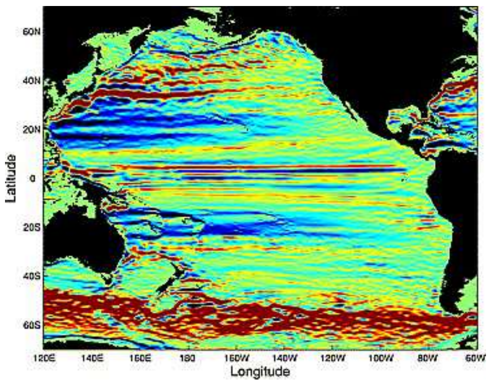


(Stephenson et al 2004)

Zonal jets revisited: Ocean currents

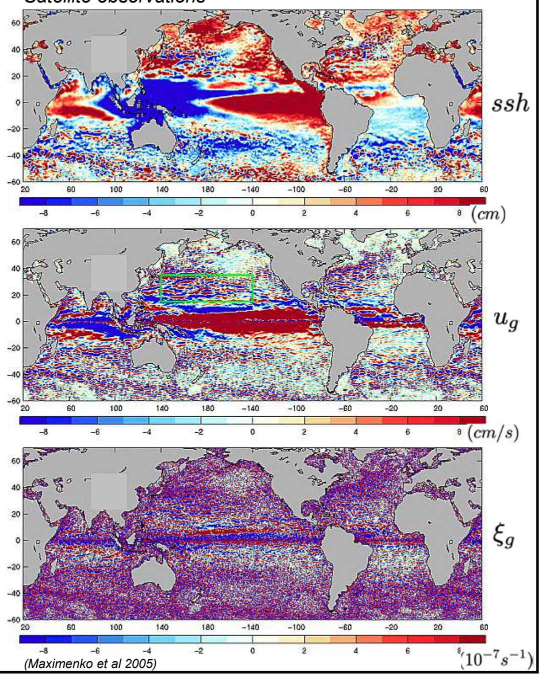
Altimetric observations and high resolution models have shown that the large scale ocean circulation on timescales of a few months is characterised by zonal jets of alternating sign.

Ocean model u_{400m}



(Richards et al 2006)

Satellite observations



(Maximenko et al 2005)

Wave-Turbulence crossover

Remember the Rossby radius ? The length scale on which relative vorticity and vortex stretching make equal contributions to potential vorticity:

$$\nabla^2\psi \sim \frac{f^2}{gH}\psi \quad \Rightarrow \quad L \sim \frac{\sqrt{gH}}{f}$$

Now let's consider larger scales. Compare advection of planetary and relative vorticity:

$$\frac{\partial \xi}{\partial t} + \mathbf{v}.\nabla\xi + \beta v = 0 \quad \rightarrow \quad \mathbf{v}.\nabla\xi \sim \beta v$$

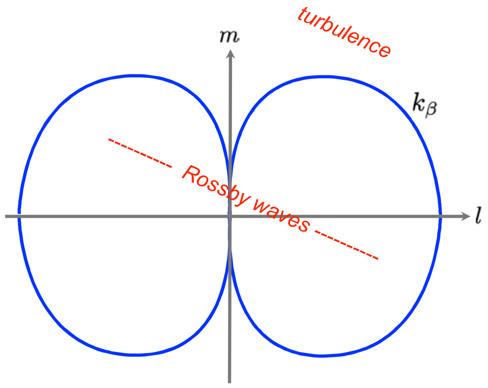
Scale analysis of this ->

$$U\frac{U}{L^2} \sim \beta U \quad \rightarrow \quad L \sim \sqrt{\frac{u}{\beta}}$$

This is called the "Rhines scale", where Rossby waves give way to turbulence.

Compare Rossby wave frequency with a typical turbulence inverse timescale

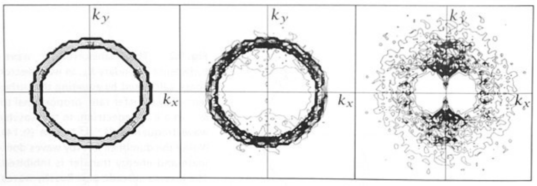
$$\omega = \frac{\beta l}{k^2} \sim u^* k \quad \rightarrow \quad k^2 = \frac{\beta}{u^*}\cos\theta$$

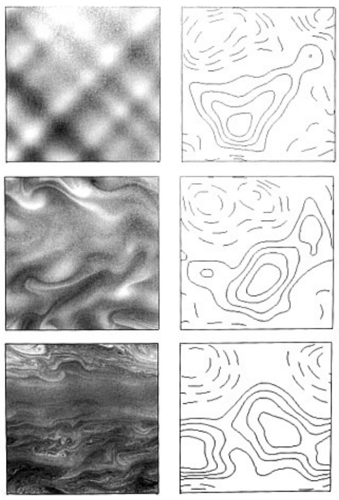


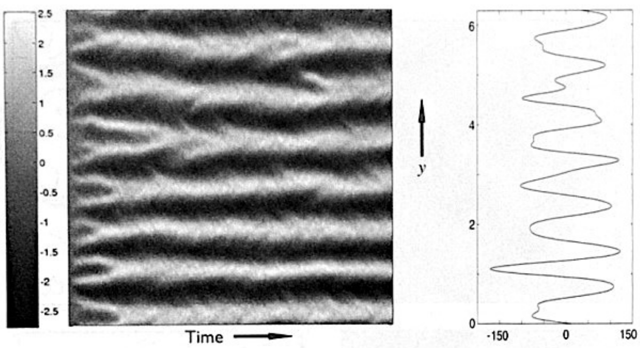
This leads to an anisotropic boundary in wavenumber space between waves and turbulence

Collapse to zonal jets

Physically, Rossby wave solutions exist inside the dumbell. Scale transfer is not possible in this region. Cascade is therefore towards $k_x = 0$, $k_y \neq 0$. This implies zonal jets separated in latitude by scale k_β.







Scale separation and boundary conditions