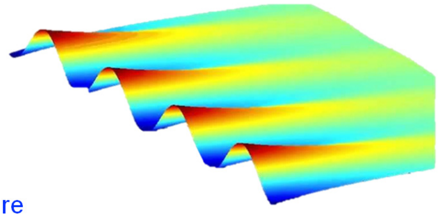
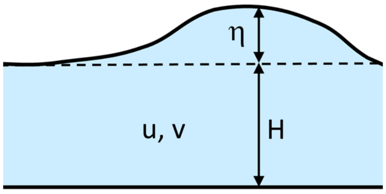
Chapter 4: Gravity waves and tropical dynamics

- ⇨ Gravity waves and rotation
- ⇨ Coastal Kelvin waves
- ⇨ Scales and approximations near the equator
- ⇨ Equatorial Kelvin waves
- ⇨ Equatorial wave solutions: dispersion and structure



Gravity waves in shallow water

Let's start with something simple: a one-dimensional non-rotating linear system.
Which terms shall we cross out ?



$$\frac{\partial u}{\partial t}+\cancel{u\frac{\partial u}{\partial x}}+\cancel{v\frac{\partial u}{\partial y}}-\cancel{fv}=-g\frac{\partial h}{\partial x}$$ x-momentum

$$\cancel{\frac{\partial v}{\partial t}+\cancel{u\frac{\partial v}{\partial x}}+\cancel{v\frac{\partial v}{\partial y}}+\cancel{fu}=-g\frac{\partial h}{\partial y}}$$ y-momentum

$$\frac{\partial h}{\partial t}+\cancel{u\frac{\partial h}{\partial x}}+\cancel{v\frac{\partial h}{\partial y}}=-\overset{H}{h}\left(\frac{\partial u}{\partial x}+\cancel{\frac{\partial v}{\partial y}}\right)$$ continuity $(h=H+\eta)$

This leaves us with

$$\frac{du}{dt}=-g\frac{dh}{dx},\quad \frac{dh}{dt}=-H\frac{du}{dx}\quad\Rightarrow\quad\frac{d^2u}{dt^2}=gH\frac{d^2u}{dx^2}$$

and the solution of this equation is $u=Re\,\tilde{u}\,e^{i(lx-\omega t)}$

a gravity wave with a simple dispersion relation $\omega^2=gHl^2,\quad c=\frac{\omega}{l}=\pm\sqrt{gH}\left(=\frac{d\omega}{dl}\right)$

Adding rotation

Put the rotation back into the linear system, we need two dimensions and three equations again:

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \eta}{\partial x}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \eta}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

substitute solution $(u, v, \eta) = (\tilde{u}, \tilde{v}, \tilde{\eta})e^{i(lx+my-\omega t)}$

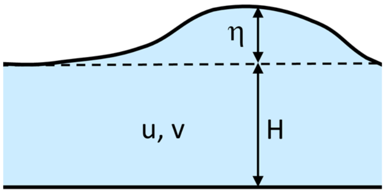
remember $\frac{\partial}{\partial x} \rightarrow il\,\times \quad \frac{\partial}{\partial y} \rightarrow im\,\times \quad \frac{\partial}{\partial t} \rightarrow -i\omega\times$

Differential equations become linear algebraic equations

$$-i\omega\tilde{u} - f\tilde{v} = -igl\tilde{\eta}$$

$$-i\omega\tilde{v} + f\tilde{u} = -igm\tilde{\eta}$$

$$-i\omega\tilde{\eta} + H(il\tilde{u} + im\tilde{v}) = 0$$



The unknowns are the wave amplitudes $\tilde{u}, \tilde{v}, \tilde{\eta}$

The parameters are the wave properties l, m, ω *and the geophysical constants* f, g, H

Inertia-gravity (Poincaré) waves

We need to solve the algebraic system

$$\begin{pmatrix} -i\omega & -f & igl \\ f & -i\omega & igm \\ ilH & imH & -i\omega \end{pmatrix} \begin{pmatrix} \tilde{u} \\ \tilde{v} \\ \tilde{\eta} \end{pmatrix} = 0$$

⇨ *Trivial solution* $\tilde{u} = \tilde{v} = \tilde{\eta} = 0$ *(no flow)*

⇨ *The condition for having non-trivial solutions is that the determinant of the matrix is zero.*

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = a \bullet \det\begin{bmatrix} e & f \\ h & i \end{bmatrix} - b \bullet \det\begin{bmatrix} d & f \\ g & i \end{bmatrix} + c \bullet \det\begin{bmatrix} d & e \\ g & h \end{bmatrix}$$

This leads to: $\omega\left[\omega^2 - f^2 - gH(l^2 + m^2)\right] = 0$

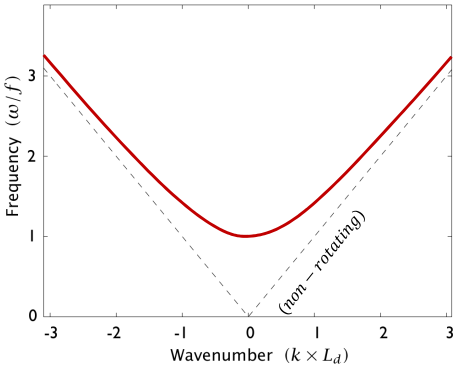
So the solutions are either $\omega = 0$ *(steady geostrophic flow)*

Or "Inertia-gravity" waves or Poincaré waves:

$$\omega = \pm\sqrt{f^2 + gHk^2}$$

- *For short waves (large l) these waves behave like ordinary, non-dispersive gravity waves.*
- *For long waves (small l) the frequency has a lower limit of f, and the waves become very dispersive, to the point where they break down into coherent but unconnected free motion in inertial circles.*

Is there any way to have a large-scale gravity wave that propagates normally on a rotating planet ? Yes !



Boundary Kelvin waves

Add a lateral boundary to the problem, cross out terms involving flow perpendicular to the boundary

$$\cancel{\frac{\partial u}{\partial t}}\boxed{-fv=-g\frac{\partial\eta}{\partial x}}$$

Geostrophic balance

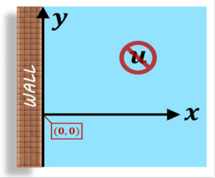
$$\boxed{\begin{aligned}&\frac{\partial v}{\partial t}+\cancel{fu}=-g\frac{\partial\eta}{\partial y}\\&\frac{\partial\eta}{\partial t}+H\left(\cancel{\frac{\partial u}{\partial x}}+\frac{\partial v}{\partial y}\right)=0\end{aligned}}$$

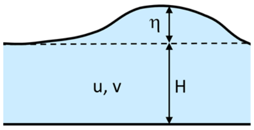
non-dispersive waves

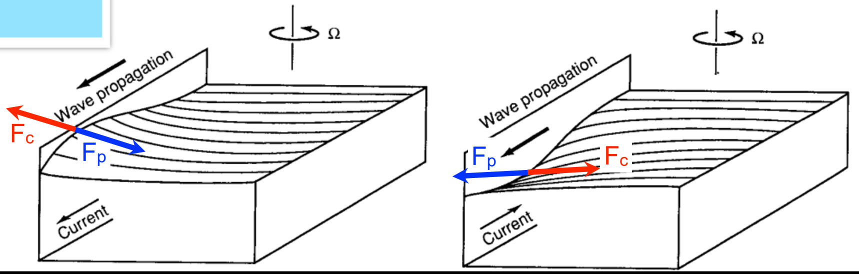
In the x direction we have geostrophic balance, with pressure and Coriolis forces alternating in direction as crests and troughs propagate meridionally.

In the y direction we have non-dispersive gravity waves with a fixed phase speed independent of horizontal scale

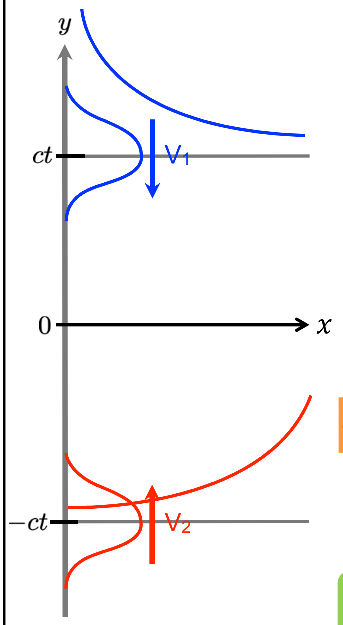
$$|c|=\sqrt{gH}$$







Boundary Kelvin waves



⇨ *Since the wave is non-dispersive, all signals must travel at speed c. The solution for v at y=0 and time t consists of two waves traveling in opposite directions:*

$$v=V_1(x,y+ct)+V_2(x,y-ct)$$

⇨ *The corresponding solution for η is* $\eta=\sqrt{H/g}\,(-V_1+V_2)$

(check this by substitution:

$$\begin{aligned}&\rightarrow\frac{\partial}{\partial t}(V_1+V_2)=-\sqrt{gH}\frac{\partial}{\partial y}(-V_1+V_2)\qquad&&\rightarrow\frac{\partial V_1}{\partial t}=c\frac{\partial V_1}{\partial y}\\&\frac{\partial}{\partial t}(-V_1+V_2)=-\sqrt{gH}\frac{\partial}{\partial y}(V_1+V_2)&&\frac{\partial V_2}{\partial t}=-c\frac{\partial V_2}{\partial y}\end{aligned}$$

)

⇨ *Substituting this solution into the geostrophic balance equation we can derive the x-dependence*

$$-fv=-g\frac{\partial\eta}{\partial x}\qquad\frac{\partial V_1}{\partial x}=-\frac{f}{\sqrt{gH}}V_1\qquad\frac{\partial V_2}{\partial x}=\frac{f}{\sqrt{gH}}V_2$$

⇨ *These relations have exponential solutions in x with a scale distance of the Rossby radius of deformation L_R = c/f.*

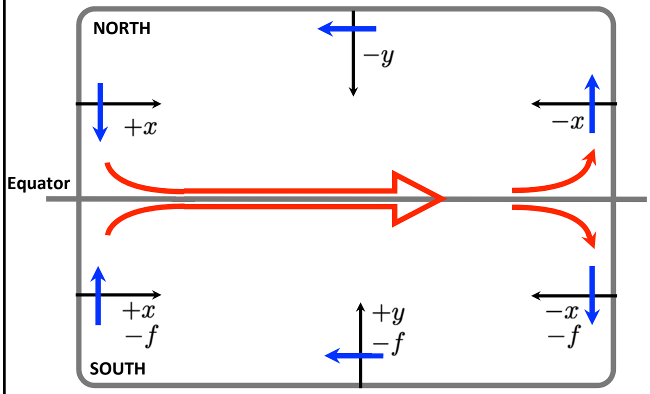
$$V_1(x,ct)e^{-\frac{x}{L_R}}\qquad V_2(x,ct)e^{\frac{x}{L_R}}$$

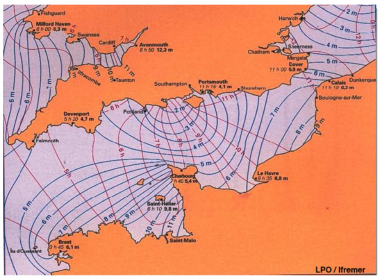
$x\geq 0$
f> 0

V_2 is a growing solution so we reject it as unphysical ($x>0$).
V_1 decays away from the coast with boundary layer width L_R

Properties of Kelvin waves

Since the only admissible solution is V_1, we conclude that for a system bounded on the west (x positive) the wave propagates in the negative y direction, i.e. to the south. If x is negative this reverses so on the eastern side of the basin the Kelvin wave goes northwards. So in the northern hemisphere a Kelvin wave will keep the coast to its right as it is pushed against it by the Coriolis force.





Tides are higher on the French side because the signal propagates in from the west

In the southern hemisphere f changes sign so all these considerations are reversed, and Kelvin waves propagate with the coast to the left.

What happens at the Equator ?
Can northern and southern Kelvin waves get pushed against each other for mutual support ?

Scales of motion near the Equator

At the equator

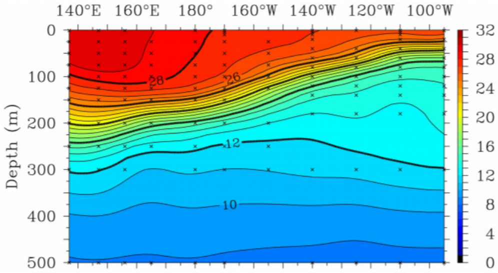
$$\phi = 0, \ \ f = 0, \ \ \beta = \frac{2\Omega}{a} = 2.28 \times 10^{-11}\, m^{-1} s^{-1}$$

β approximation: $f = \beta y$

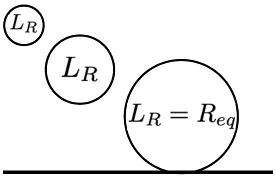
⇨ Consider a single layer overlying the abyss.
The internal Rossby radius is

$$L_R = \frac{\sqrt{g'H}}{f} = \frac{c}{f}$$

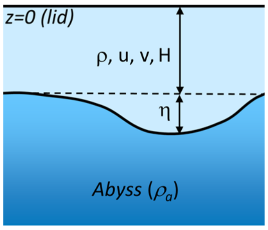
⇨ How does this work at the equator where f=0 ?



$\Delta\rho/\rho = 0.002$ ($g' = g\,\Delta\rho/\rho$), $H = 100$m
Gives gravity wave speed $c = 1.4$ m/s



$$L_R = \frac{c}{\beta R_{eq}}$$



↳ Define equatorial radius of deformation $R_{eq} = \sqrt{\frac{c}{\beta}} \sim 250\text{km} \ \ \sim 2.2^0$

↳ The time T_{eq} for a wave to travel distance R_{eq} $T_{eq} = \frac{1}{\sqrt{\beta c}} \sim 2$ days

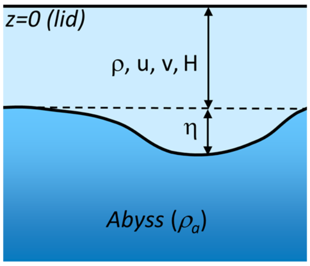
Linear Equatorial shallow water model

⇨ Consider linear perturbations on a resting basic state

$$\frac{\partial u}{\partial t} - \beta y v = -g'\frac{\partial \eta}{\partial x}$$

$$\frac{\partial v}{\partial t} + \beta y u = -g'\frac{\partial \eta}{\partial y}$$

$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$



⇨ First we'll look for a special case - with v = 0

The equatorial Kelvin wave solution

Assume no meridional flow

$$v = 0$$

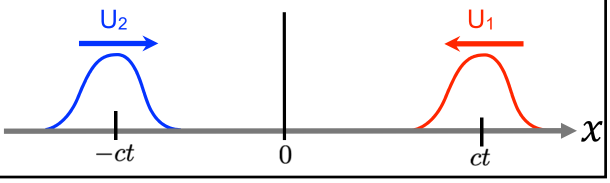
non-dispersive
Waves
$c = \sqrt{g'H}$

$$\boxed{\begin{aligned}\frac{\partial u}{\partial t} - \cancel{\beta y v} &= -g'\frac{\partial \eta}{\partial x}\\ \frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}\right) &= 0\end{aligned}}$$

$$\cancel{\frac{\partial v}{\partial t}} + \boxed{\beta y u = -g'\frac{\partial \eta}{\partial y}}$$

Cross-equatorial geostrophic balance

As before, this is a wave equation that has non-dispersive solutions with wave speed c for all wavenumbers. So any function of x will translate at speed c. The solution at x can be any function of (x ± ct).



The Kelvin wave solution

Assume no meridional flow

$$v = 0$$

non-dispersive
Waves
$c = \sqrt{g'H}$

$$\boxed{\begin{aligned} &\frac{\partial u}{\partial t} - \cancel{\beta y v} = -g' \frac{\partial \eta}{\partial x} \\ &\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \cancel{\frac{\partial v}{\partial y}}\right) = 0 \end{aligned}}$$

$$\cancel{\frac{\partial v}{\partial t}} + \boxed{\beta y u = -g' \frac{\partial \eta}{\partial y}}$$

Cross-equatorial geostrophic balance

As before, this is a wave equation that has non-dispersive solutions with wave speed c for all wavenumbers. So any function of x will translate at speed c. The solution at x can be any function of (x ± ct).

As for coastal Kelvin waves, we can postulate solutions of the form the superposition of 2 independant waves

$$u = U_1(x + ct) + U_2(x - ct)$$

where ***U_1*** propagates westwards and ***U_2*** propagates eastwards

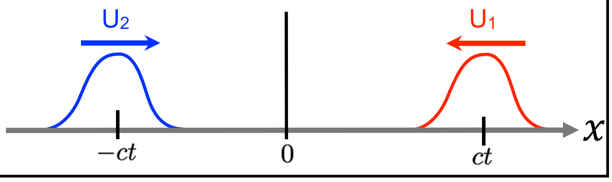
As before, the solution for η can be written in terms of U_1 and U_2

$$\eta = \sqrt{\frac{H}{g'}}(-U_1 + U_2)$$

(which can be verified by substitution, to give

$$\frac{\partial U_1}{\partial t} = c \frac{\partial U_1}{\partial x}$$

$$\frac{\partial U_2}{\partial t} = -c \frac{\partial U_2}{\partial x} \quad)$$



The EKW wave properties

⇨ The meridional structure is given by the remaining equation which expresses cross-equatorial geostrophic balance !

$$\boxed{\beta y u = -g' \frac{\partial \eta}{\partial y}}$$

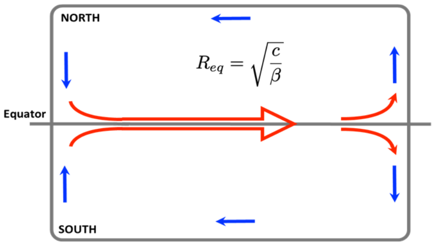
⇨ Substituting our solutions gives:

$$\beta y(U_1 + U_2) = -c \frac{\partial}{\partial y}(-U_1 + U_2)$$

Meridional structures are: $U_1 \sim e^{\frac{\beta}{2c} y^2}$ $\quad U_2 \sim e^{-\frac{\beta}{2c} y^2}$

↳ Only the **eastward propagating** solution U_2 is exponentially decaying in y^2. Note the difference with coastal waves that depended on nonzero f, and thus, y. Now we have a y^2 dependence that works both to the north and south with the same propagation direction.

⇨ If we write $U_2(x – ct) = c\psi(x – ct)$, where ψ is a dimensionless wave form in the x-direction, **equatorial Kelvin wave solution** can be written:



$$\begin{aligned} &u = c\,\psi(x - ct)\, e^{-y^2/2R_{eq}^2} \\ &v = 0 \\ &\eta = H\,\psi(x - ct)\, e^{-y^2/2R_{eq}^2} \end{aligned}$$

EKW have the following properties:
- propagates eastwards
- non-dispersive $\quad c = \sqrt{g'H}$
- maximum on equator

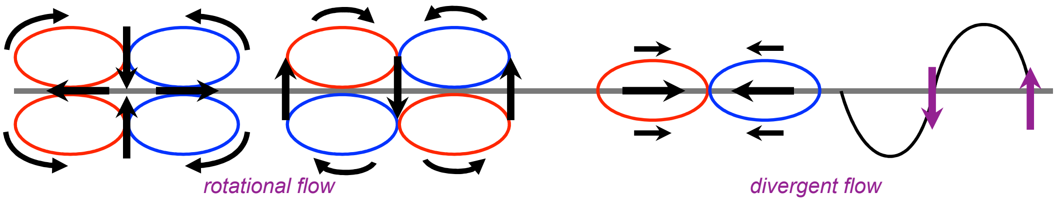
Cf. coastal Kelvin wave, propped up against the coast.
An EKW is "propped up" against another equatorial Kelvin wave.

The general solution

Now we allow wavelike variations in the zonal direction including for v

$$u = \tilde{u}(y)e^{i(lx-wt)} \qquad v = \tilde{v}(y)e^{i\left(lx-wt+\frac{\pi}{2}\right)} \qquad \eta = \tilde{\eta}(y)e^{i(lx-wt)}$$

Note that we specify u and η in phase with one another, but *<u>v is in quadrature with them</u>*.



Substitution into equatorial shallow water equations …

$$\frac{\partial u}{\partial t} - \beta y v = -g'\frac{\partial \eta}{\partial x}$$
$$\frac{\partial v}{\partial t} + \beta y u = -g'\frac{\partial \eta}{\partial y}$$
$$\frac{\partial \eta}{\partial t} + H\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = 0$$

details

$$\begin{cases} \dfrac{\partial u}{\partial t} - \beta y v = -g'\dfrac{\partial \eta}{\partial x} \\ \dfrac{\partial v}{\partial t} + \beta y u = -g'\dfrac{\partial \eta}{\partial y} \\ \dfrac{\partial \eta}{\partial t} + H\left(\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y}\right) = 0 \end{cases}$$

$$u = \tilde{u}(y)e^{i(kx-\omega t)} \qquad v = \tilde{v}(y)e^{i(kx-\omega t\pm\pi/2)}$$
$$\eta = \tilde{\eta}(y)e^{i(kx-\omega t)} \qquad = \tilde{v}(y)e^{i(kx-\omega t)}e^{\pm i\pi/2}$$
$$= \pm i\tilde{v}(y)e^{i(kx-\omega t)}$$

v in quadrature with u,
+ or - makes no difference, we choose +

$$\begin{cases} -i\omega\tilde{u} - i\beta y\tilde{v} + ig'k\tilde{\eta} = 0 \\ \omega\tilde{v} + \beta y\tilde{u} + g'\dfrac{\partial\tilde{\eta}}{\partial y} = 0 \\ -i\omega\tilde{\eta} + H\left(ik\tilde{u} + i\dfrac{\partial\tilde{v}}{\partial y}\right) = 0 \end{cases}$$

We want to eliminate u and η to get an equation for v.

We drop tildes and prime on g, and we use subscript notation for derivatives. The linear system can be written:

$$\begin{cases} \omega u + \beta y v - gk\eta = 0 & (1) \\ \omega v + \beta y u + g\dfrac{\partial \eta}{\partial y} = 0 & (2) \\ -\omega\eta + Hku + H\dfrac{\partial v}{\partial y} = 0 & (3) \end{cases}$$

details

$$\left\{\begin{array}{rl} \partial/\partial y(1) + k\times(2) \rightarrow \omega u_y + \beta v + \beta y v_y + \omega k v + \beta y k u = 0 & (A) \\ \omega\times(2) + g\times\partial/\partial y(3) \rightarrow \omega^2 v + \beta y \omega u + gHku_y + gHv_{yy} = 0 & (B) \\ \omega\times(1) - gk\times(3) \rightarrow \omega^2 u + \beta y \omega v - gk^2 Hu - gkHv_y = 0 & (C) \end{array}\right.$$

⇨ $gHk\times(A) - \omega\times(B) \rightarrow$

$$gHk(\beta v + \beta y v_y + \omega k v) + gHk^2\beta y u - \omega^3 v - \beta y \omega^2 u - gH\omega v_{yy} = 0$$

$$-gH\omega v_{yy} + gHk\beta y v_y + (gHk\beta + gH\omega k^2 - \omega^3)v + (gHk^2\beta y - \beta y \omega^2)u = 0 \qquad (D)$$

⇨ $(D) + \beta y\times(C) \rightarrow$

$$-gH\omega v_{yy} + \cancel{gHk\beta y v_y} + (gHk\beta + gH\omega k^2 - \omega^3)v + \beta^2 y^2 \omega v - \cancel{\beta y g H k v_y} = 0$$

⇨ $\div - gH\omega \rightarrow \dfrac{d^2\tilde{v}}{dy^2} + \left[\dfrac{\omega^2}{g'H} - k^2 - \dfrac{k\beta}{\omega} - \dfrac{\beta^2}{g'H}y^2\right]\tilde{v} = 0$

or $\dfrac{d^2\tilde{v}}{dy^2} + \dfrac{\beta^2}{c^2}(Y^2 - y^2)\tilde{v} = 0$ where $\left\{\begin{array}{l} c \text{ is the gravity wave speed)} \\ Y^2 = \dfrac{g'H}{\beta^2}\left[\dfrac{\omega^2}{g'H} - k^2 - \dfrac{k\beta}{\omega}\right] \end{array}\right.$

The general solution

Now we allow wavelike variations in the zonal direction including for v

$u = \tilde{u}(y)e^{i(lx-wt)}$ $\qquad v = \tilde{v}(y)e^{i\left(lx-wt+\frac{\pi}{2}\right)}$ $\qquad \eta = \tilde{\eta}(y)e^{i(lx-wt)}$

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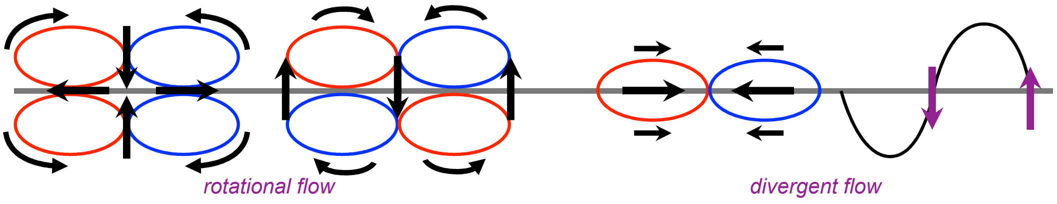


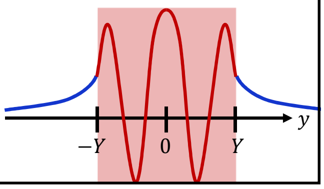
Substitution into equatorial shallow water equations gives

$$\frac{d^2\tilde{v}}{dy^2} + \frac{\beta^2}{c^2}(Y^2 - y^2)\tilde{v} = 0 \quad \textit{where} \quad Y^2 = \left(\frac{\omega^2}{c^2} - l^2 - \frac{\beta l}{\omega}\right)\frac{c^2}{\beta^2}$$

y < Y : oscillating solutions in y
y > Y : decaying solutions in y

Y is the width of the "equatorial waveguide".

Y depends on wavelength and frequency but scales similar to R_{eq}.
It represents the zone in which we have some meridional wave structure.
Outside this zone the amplitude decays exponentially with latitude.



Meridional structure

It can be shown that the general solution is of the form

$$\tilde{v} \propto H_n(y')\ e^{-y'^2/2} \qquad (y' = y/R_{eq})$$

Remember that u and η have opposite symmetry to v

and substitution of this form into the differential equation for v leads to the dispersion relation

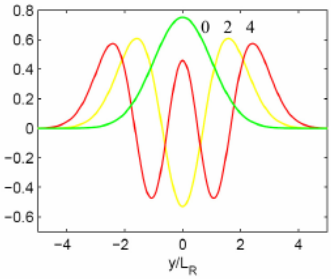
$$\frac{\omega^2}{c^2} - l^2 - \frac{\beta l}{\omega} = (2n+1)\frac{\beta}{c} = \frac{(2n+1)}{R_{eq}^2}$$

In fact this is a set of dispersion relations corresponding to a discrete set of meridional structures $H_n(y')$, the "Hermite polynomials".

$$H_0(y') = 1$$
$$H_1(y') = 2y'$$
$$H_2(y') = 4y'^2 - 2$$
$$H_3(y') = 8y'^3 - 12y'$$
$$H_4(y') = 16y'^4 - 48y'^2 - 12 \quad \ldots$$

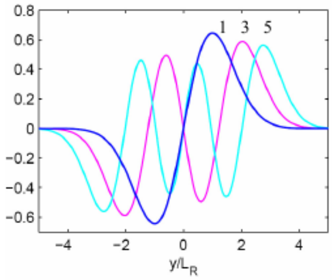
$$y' H_n = nH_{n-1} + \frac{1}{2}H_{n+1} \qquad \frac{dH_n}{dy'} = 2nH_{n-1}$$

Symmetric structures for v: n=0,2,4...



Cross-equatorial flow and anti-symmetric η

Anti-symmetric structures for v: n=1,3,5...



No cross-equatorial flow (convergence/divergence) and symmetric thermocline displacements

details

$$\frac{d^2v}{dy^2} + \frac{1}{R_{eq}^4}(Y^2 - y^2)v = 0$$

$$R_{eq} = \sqrt{\frac{c}{\beta}}, \quad Y^2 = \left(\frac{\omega^2}{c^2} - k^2 - \frac{\beta k}{\omega}\right)R_{eq}^4 \quad \{= (2n+1)R_{eq}^2\}$$

$$y' = y/R_{eq}, \quad Y' = Y/R_{eq} \rightarrow \quad \frac{1}{R_{eq}^2}\frac{d^2v}{dy'^2} + \frac{1}{R_{eq}^4}(Y'^2 - y'^2)vR_{eq}^2 = 0$$

dropping primes $\frac{d^2v}{dy^2} + (Y^2 - y^2)v = 0$ solution $v = H_n e^{-y^2/2}$

should lead to non-dimensional dispersion relation $Y^2 = 2n+1$

using $\frac{dH_n}{dy} = 2nH_{n-1}$ and $yH_n = nH_{n-1} + \frac{H_{n+1}}{2}$

details

$$\frac{dv}{dy} = \frac{dH_n}{dy} e^{-y^2/2} - yH_n e^{-y^2/2} = \left[\frac{dH_n}{dy} - yH_n\right] e^{-y^2/2}$$

$$\frac{dv}{dy} = \left[2nH_{n-1} - \left(nH_{n-1} + \frac{H_{n+1}}{2}\right)\right] e^{-y^2/2} = \left[nH_{n-1} - \frac{H_{n+1}}{2}\right] e^{-y^2/2} = \left[yH_n - H_{n+1}\right] e^{-y^2/2}$$

$$\frac{d^2v}{dy^2} = \left[H_n + y\frac{dH_n}{dy} - \frac{dH_{n+1}}{dy} - y\left(yH_n - H_{n+1}\right)\right] e^{-y^2/2}$$

$$= \left[H_n + 2ynH_{n-1} - 2(n+1)H_n - y^2H_n + yH_{n+1}\right] e^{-y^2/2}$$

$$= \left[H_n + y\left(2nH_{n-1} - yH_n + H_{n+1}\right) - 2(n+1)H_n\right] e^{-y^2/2}$$

$$= \left[-H_n - 2nH_n + y^2H_n\right] e^{-y^2/2}$$

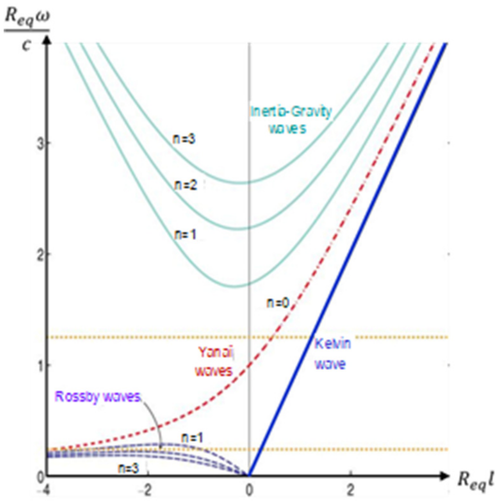
so $\left[y^2 - (2n+1)\right] H_n e^{-y^2/2} + (Y^2 - y^2) H_n e^{-y^2/2} = 0$

thus $Y^2 = 2n + 1$

The dispersion relations

⇨ Substitution of the general solutions into the differential equation for v leads to a set of dispersion relations: $\frac{\omega^2}{c^2} - l^2 - \frac{\beta l}{\omega} = (2n+1)\frac{\beta}{c}$ ⇨ *There are 3 roots for each value of n ≥ 1.*

⇨ The entire family of equatorially trapped waves: $\omega = \omega(l, n)$



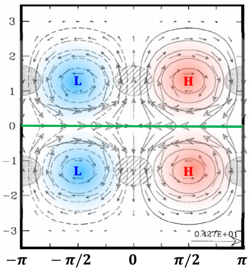
- *The largest roots are for high frequencies ($T < T_{eq}$). ☞ They are* ***inertia-gravity waves*** *slightly modified by the beta effect.*
- *The smaller root for ω are equatorial* ***Rossby waves***
- *A mixed Rossby / Inertia Gravity wave (sometimes called "Yanai wave") exists for n = 0.*
- *The special case of v = 0 corresponds to n = –1, this is the Kelvin wave.*

Wave properties

⇨ *Odd order waves (n = –1,1,3..) are symmetric in η : Kelvin, Rossby and Inertia Gravity waves.*
⇨ *Even order waves (n = 0,2...) are antisymmetric in η : mixed Rossby-Gravity waves*

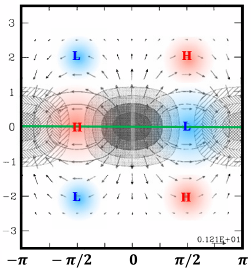
$n = 1,\ l^* = 1$

Equatorial Rossby wave



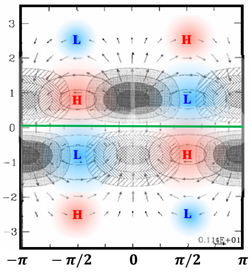
$n = 1, l^* = 1$

Westward IG wave



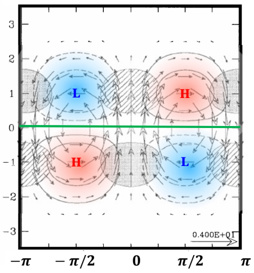
$n = 2, l^* = 1$

Eastward IG wave



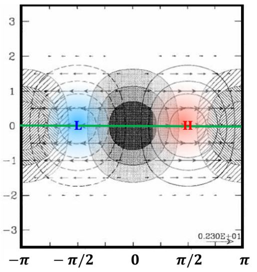
$n = 0, l^* = 1$

Mixed Rossby-gravity wave



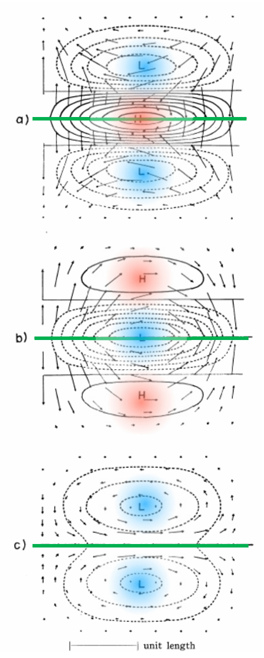
$n = -1, l^* = 1$

Equatorial Kelvin wave



n≥1 Equatorial wave structures

Fig. 4. Pressure and velocity distributions of eigensolutions for $n=1$
a: Eastward propagating inertio-gravity wave
b: Westward propagating inertio-gravity wave
c: Rossby wave.
from Matsuno (1966)



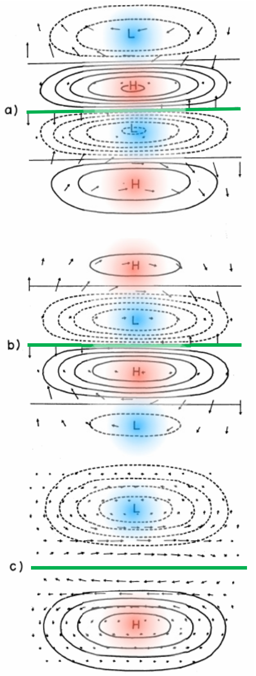


Fig. 5. Same as Fig. 4 but for $n=2$.
from Matsuno (1966)

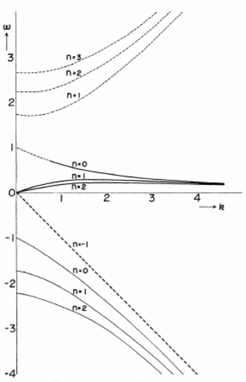


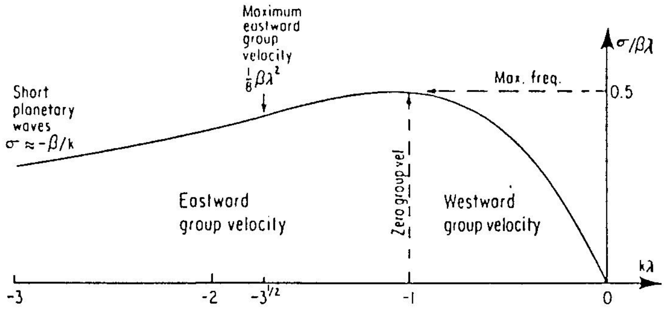
Fig. 3a. Frequencies as functions of wave number.
Thin solid line: eastward propating inertio-gravity waves.
Thin dashed line: westward propagating inertio-gravity waves.
Thick solid line: Rossby (quasi-geostrophic) waves.
Thick dashed line: The Kelvine wave like wave.

Equatorial Rossby waves

$$\frac{\omega^2}{c^2} - l^2 - \frac{\beta l}{\omega} = \frac{2n+1}{R_{eq}^2}$$

At low frequencies ω << f

$$\omega = -\frac{\beta l}{l^2 + (2n+1)R_{eq}^{-2}}$$



=> k negative, Rossby waves have westward phase propagation.
But the group velocity depends on the wavelength.

In practice the shorter Rossby waves with eastward group propagation are of little importance because they are dispersive, slow, and tend to dissipate.

Equatorial Rossby rays

Generally as a wave propagates its dispersion relation changes.

This is because it may change latitude, and f enters into the dispersion relation.
We will consider that f is "slowly varying". The direction of the group velocity is given by

$$\frac{dx}{dy} = \frac{\partial\omega/\partial n}{\partial\omega/\partial k} = -\frac{2\omega}{\beta}\left(-\frac{\beta k}{\omega} - \frac{\beta^2 y^2}{c^2}\right)^{\frac{1}{2}} \qquad \Rightarrow y = -\left(\frac{c^2 k}{\omega\beta}\right)^{\frac{1}{2}} \cos\left(\frac{2\omega}{c}x + \theta_0\right)$$

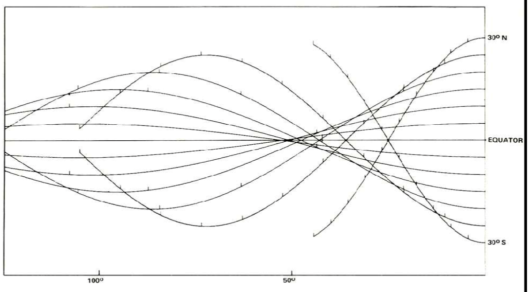
(for long R-waves)

Waves of constant frequency and zonal wavenumber will change their meridional wavenumber and thus their direction of propagation.

- they end up oscillating about the equator by refraction

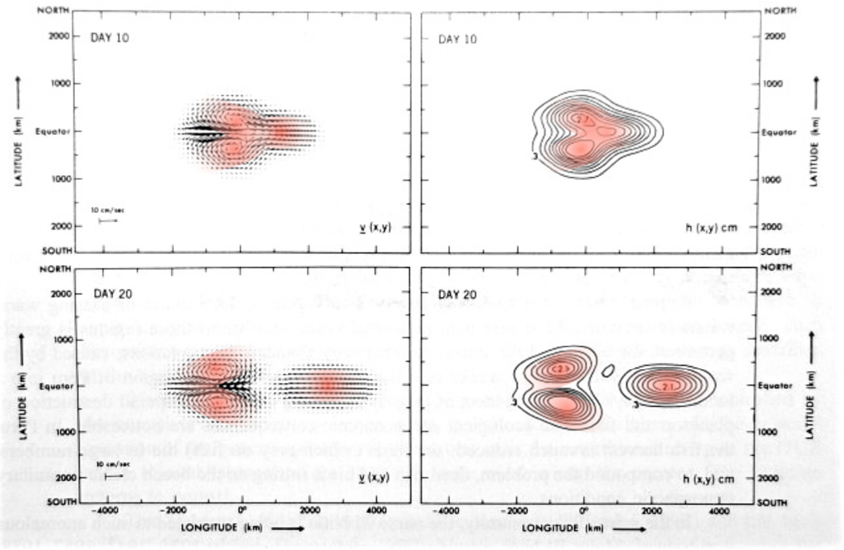
- its another way to show that they are "equatorially trapped"

- this behaviour is modified by the presence of mean currents



Oceanic adjustment

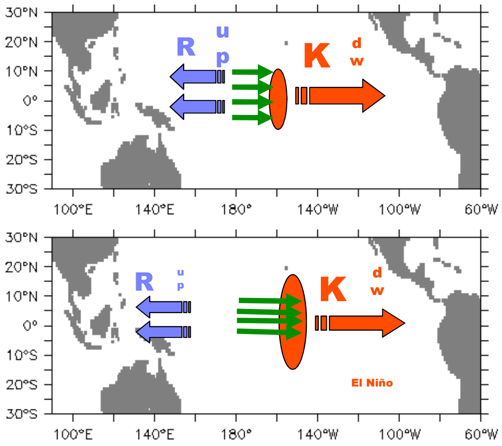
An abrupt change in the wind forcing can generate waves.
In this experiment an initial bell shaped perturbation to the thermocline is allowed to dissipate in a shallow water model. We see the single bulge (n = –1) Kelvin wave propagating eastwards and the double bulge (n = 1) Rossby wave propagating westwards.



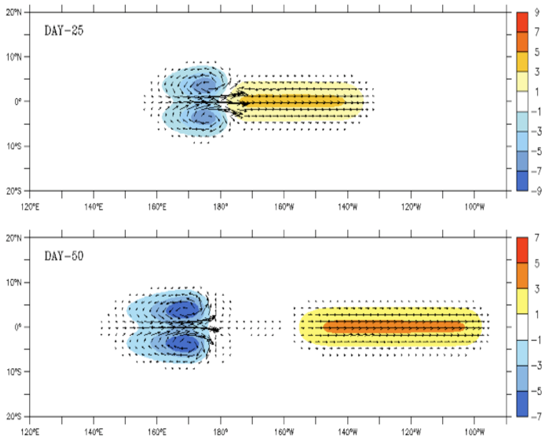
ENSO theories: the delayed oscillator

A mechanism proposed to explain how El Niño can cancel itself out the following season.
Depends on wave reflection at boundaries.

SCEMATIC

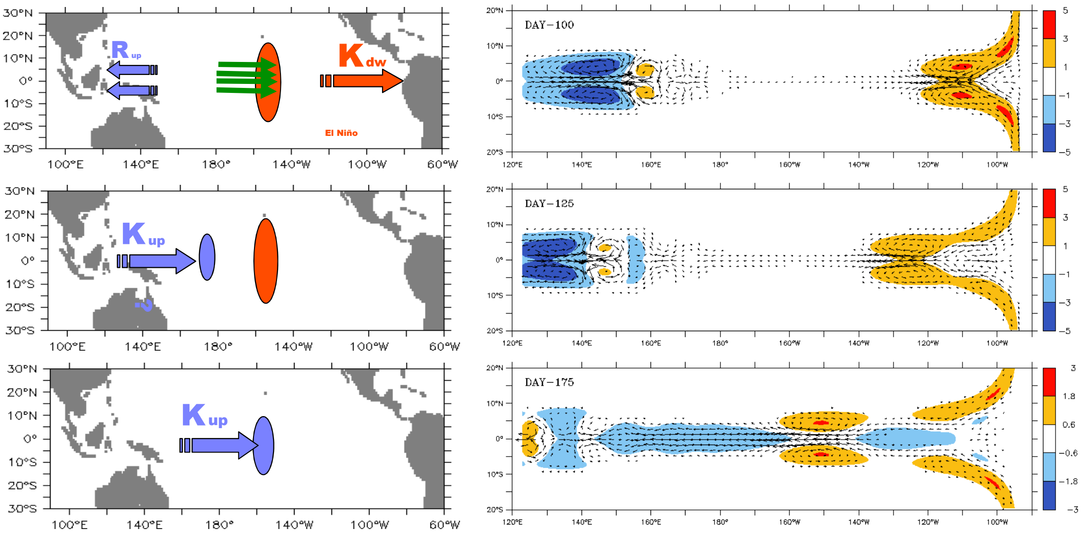


SURFACE CURRENTS and THERMOCLINE DEPTH



ENSO theories: the delayed oscillator

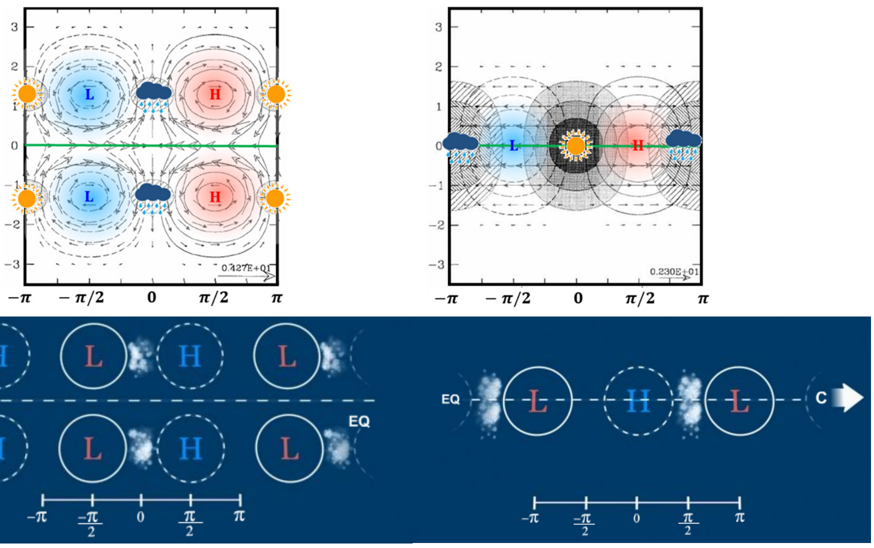
The upwelling Rossby wave at the base of the thermocline becomes an upwelling Kelvin wave traveling the other way. In the meantime, the original Kelvin wave leaks energy away at the eastern side of the basin through coastal Kelvin waves



Tropical convection in the atmosphere

☀ ⇨ Divergence ➤ downward motion ➤ evaporation ➤ clear sky ☀

🌧 ⇨ Convergence ➤ upward motion ➤ condensation ➤ clouds 🌧



$n = 1,\ l^* = 1$

Equatorial Rossby wave

$n = -1, l^* = 1$

Equatorial Kelvin wave

Tropical convection in the atmosphere

Wheeler-Kiladis space-time variance spectra