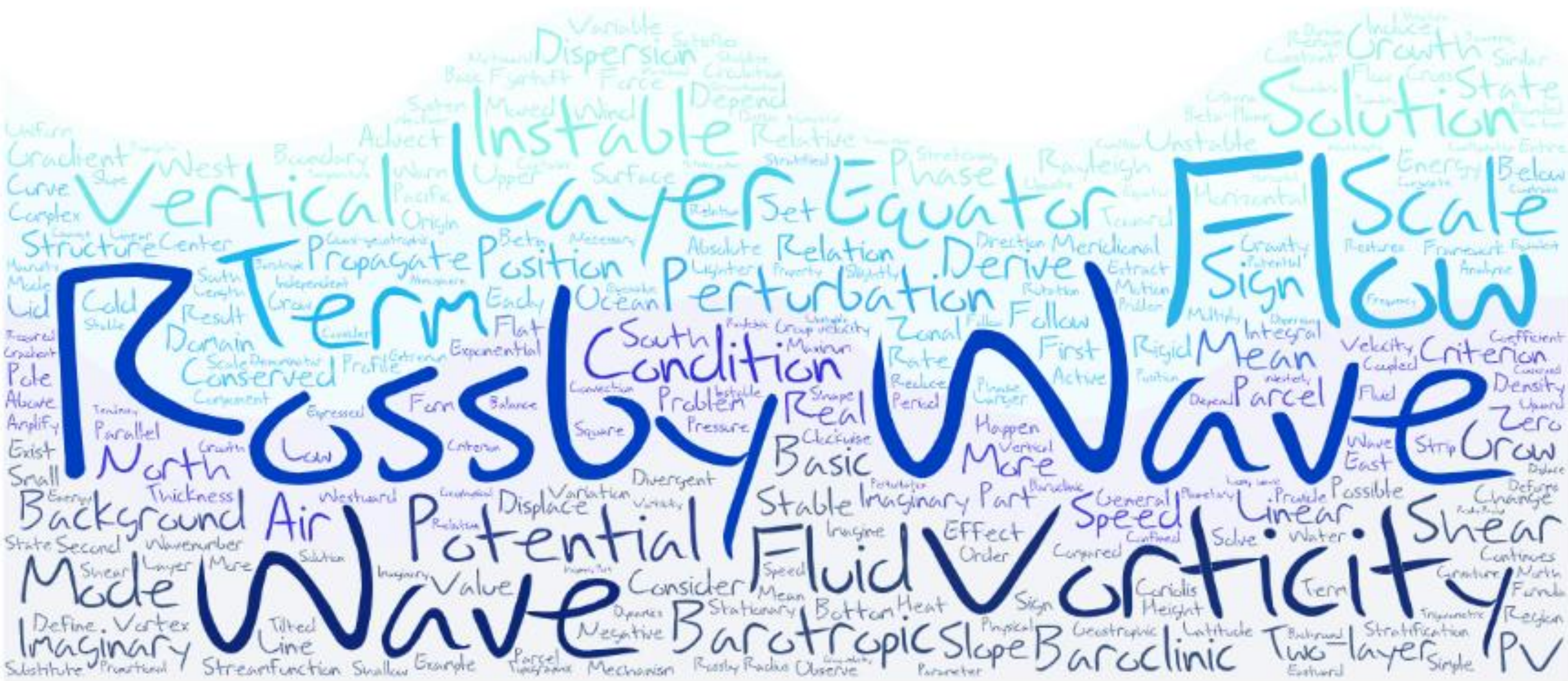
Rossby wave and instability



Exercise

Question 2

A barotropic atmospheric Rossby wave propagates westwards at 45ºN. It has global meridional scale and a zonal wavelength of 5000 km (the radius of the earth is 6400 km).

1) What is the phase speed relative to the prevailing wind ?
2) How long does the wave take to go round the world ?
3) Does this result depend on the latitude ?
4) How long would it take if the meridional scale were the same as the zonal scale ?

In the ocean at the same latitude, the thermocline is 500m deep and the difference in density between the thermocline water and the abyss is 4 kg/m^3 (the density of sea water is 1027 kg/m^3).

1) What is the westward phase speed for a Rossby wave on the thermocline with zonal and meridional wavelength of 200 km (assuming no zonal current) ?
2) How long would this wave take to cross the Pacific between 130ºW and 150ºE ?
3) What is the fastest possible transit time for very large scale waves ?

Recap: Dispersion relations of Rossby waves

CASE1: Non-divergent barotropic

$$q = \beta y + \nabla^2 \psi$$

$$c = \frac{\omega}{l} = U - \frac{\beta}{l^2 + m^2}$$

CASE2: Divergent

$$q = \beta y + \nabla^2 \psi - L_R^{-2} \psi$$

$$c = \frac{\omega}{l} = U - \frac{\beta + L_R^{-2} U}{l^2 + m^2 + L_R^{-2}}$$

CASE3: 2 active layers

H_1 $$q_1 = \beta y + \nabla^2 \psi_1 - L_1^{-2}(\psi_1 - \psi_2)$$

H_2 $$q_2 = \beta y + \nabla^2 \psi_2 + L_2^{-2}(\psi_1 - \psi_2)$$

The solution is the sum of a barotropic and a baroclinic mode

CASE4: Continuously stratified

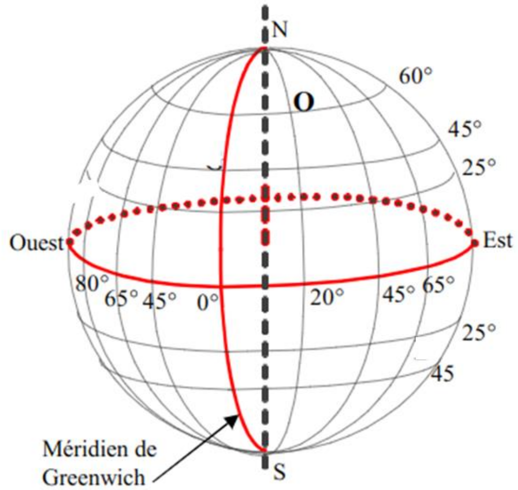
$$\frac{\partial \psi}{\partial z} = 0$$

$$q = \beta y + \nabla^2 \psi + \frac{1}{\rho_s} \frac{\partial}{\partial z} \left(\frac{f_0^2}{N^2} \rho_s \frac{\partial \psi}{\partial z} \right)$$

$$\frac{\partial \psi}{\partial z} = 0$$

The solution is the sum of a barotropic and n baroclinic modes

A barotropic atmospheric Rossby wave



45°N

$$q = \beta y + \nabla^2 \psi$$

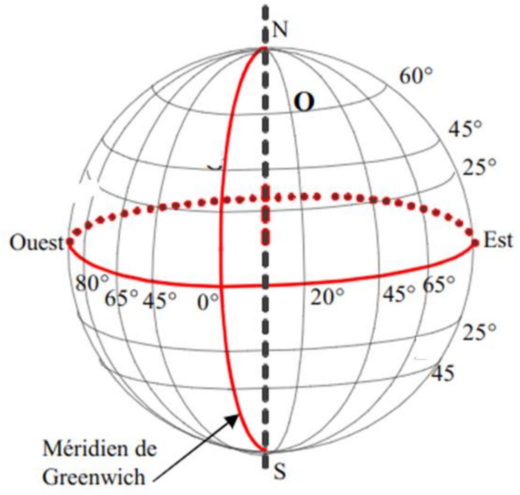
1) What is the phase speed relative to the prevailing wind ?

$$c = \frac{\omega}{l} = U - \frac{\beta}{l^2 + m^2}$$

$$c - U = -\frac{\beta}{l^2 + m^2}$$

- 45°N: $\beta = \frac{2\Omega cos\phi}{r} = \frac{2\times 2\pi \times cos(45°)}{24\times 3600\times 6400\times 10^3} = 1.61 \times 10^{-11} s^{-1}.m^{-1}$
- Global meridional scale $\Longrightarrow m = 0$ and $l = \frac{2\pi}{\lambda_x}$
- $c - U = -\frac{\beta}{l^2+m^2} = -\frac{\beta}{l^2} = -\frac{2\Omega cos\phi}{rl^2} =$ - $1.61 \times 10^{-11} \left(\frac{\lambda_x}{2\pi}\right)^2 = -10.2\ m.s^{-1}$

A barotropic atmospheric Rossby wave



45°N

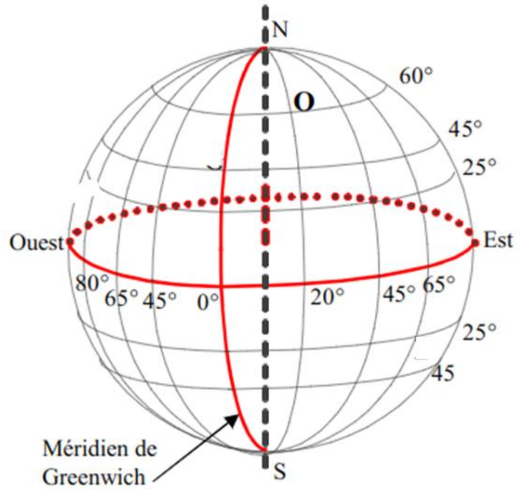
$$q = \beta y + \nabla^2 \psi$$

2) How long does the wave take to go round the world ?

$$c - U = -10.2\, m.s^{-1}$$

- 45°N: $\mathrm{d} = 2\pi r cos\phi = 2\pi \times 6400 \times cos(45°) = 28434\ km$
- $\mathrm{t} = \frac{d}{c} = 32\ days$

A barotropic atmospheric Rossby wave



45°N

$$q = \beta y + \nabla^2 \psi$$

3) Does this result depend on the latitude ?

- $t = \frac{d}{c} = \frac{2\pi r cos\phi}{\frac{\beta}{l^2}} = \frac{l^2 \times 2\pi r cos\phi}{\frac{2\Omega cos\phi}{r}} = \left(\frac{2\pi}{\lambda_x}\right)^2 \times \frac{r^2 \times \pi}{\Omega}$

A barotropic atmospheric Rossby wave

45°N

$$q = \beta y + \nabla^2 \psi$$

4) How long would it take if the meridional scale were the same as the zonal scale ?

- $c_{WEST} = \frac{\beta}{l^2 + m^2}$ with $m = l$ $\Rightarrow$ $c_2 = \frac{\beta}{2 \times l^2} = \frac{c}{2}$
- $t_2 = \frac{d}{c_2} = 2 \times \frac{d}{c} = 2 \times t = 64\ days$

Exercise

Question 2

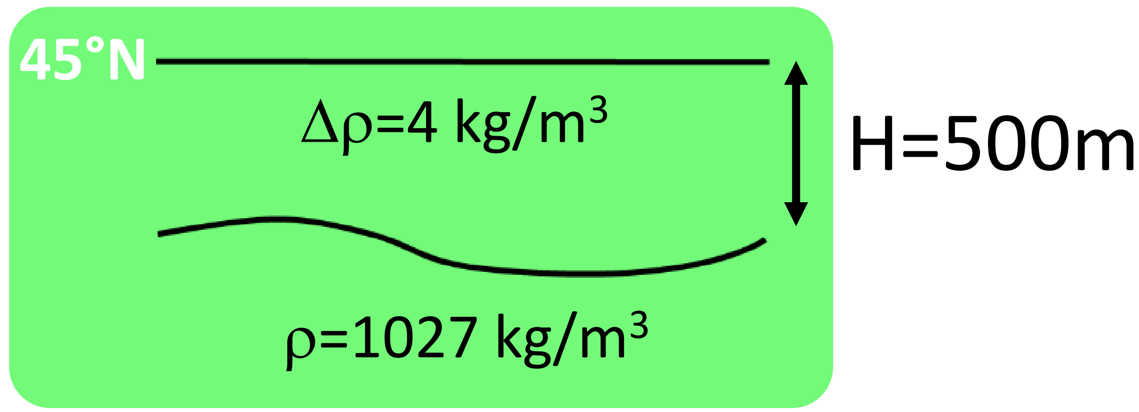
A barotropic atmospheric Rossby wave propagates westwards at 45ºN. It has global meridional scale and a zonal wavelength of 5000 km (the radius of the earth is 6400 km).

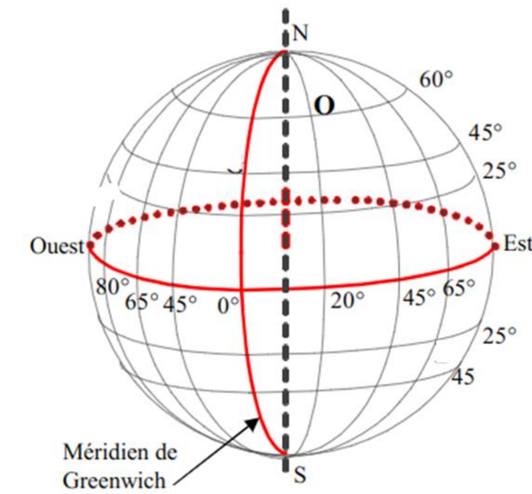
1) What is the phase speed relative to the prevailing wind ?
2) How long does the wave take to go round the world ?
3) Does this result depend on the latitude ?
4) How long would it take if the meridional scale were the same as the zonal scale ?

In the ocean at the same latitude, the thermocline is 500m deep and the difference in density between the thermocline water and the abyss is 4 kg/m^3 (the density of sea water is 1027 kg/m^3).

1) What is the westward phase speed for a Rossby wave on the thermocline with zonal and meridional wavelength of 200 km (assuming no zonal current) ?
2) How long would this wave take to cross the Pacific between 130ºW and 150ºE ?
3) What is the fastest possible transit time for very large scale waves ?

A baroclinic ocean Rossby wave



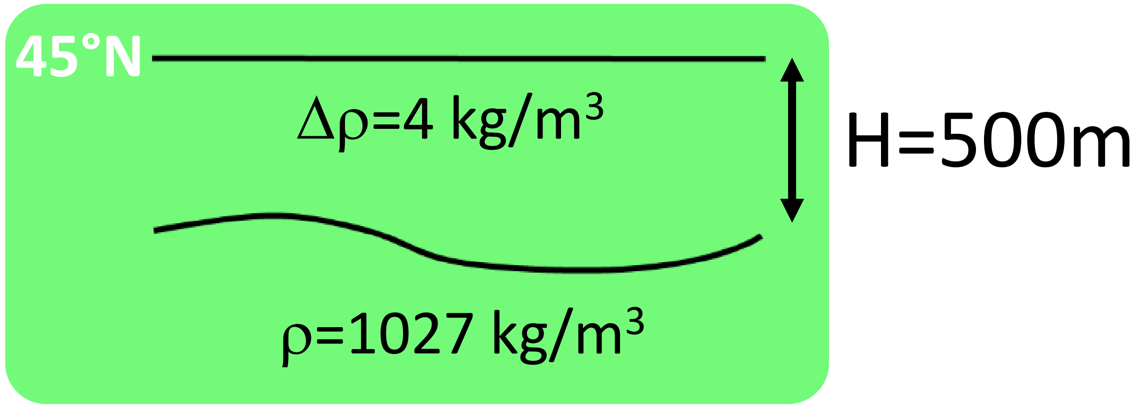


1) What is the westward phase speed for a Rossby wave on the thermocline with $\lambda_x = \lambda_y = 200\ km$ (assuming no zonal current) ?

$$\omega = -\frac{\beta l}{l^2 + m^2 + L_R^{-2}}$$

- 45°N: $f = 2\Omega sin\phi = 2 \times \frac{2\pi}{24\times3600} sin(45°) = 1.03 \times 10^{-4} s^{-1}$ and $\beta = 1.61 \times 10^{-11} s^{-1}.m^{-1}$
- $L_R = \frac{\sqrt{g'H}}{f}$ with $g' = g\frac{\Delta\rho}{\rho} = \frac{9.81\times4}{1027} = 0.0382$ and $H = 500 \Rightarrow L_R = 42km$ and $L_R^{-2} = 5.55 \times 10^{-10}$
- $c_{WEST} = \frac{\beta}{l^2+m^2+L_R^{-2}} = \frac{\beta}{2\times\left(\frac{2\pi}{\lambda_x}\right)^2 + L_R^{-2}} = 6.37 \times 10^{-3}\ m.s^{-1}$

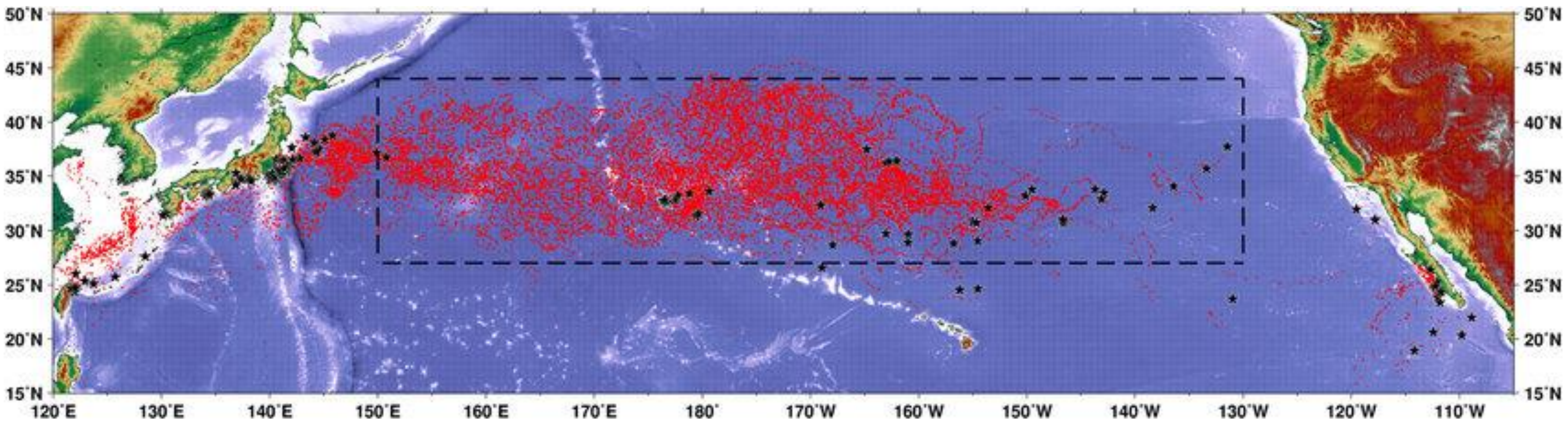
A baroclinic ocean Rossby wave



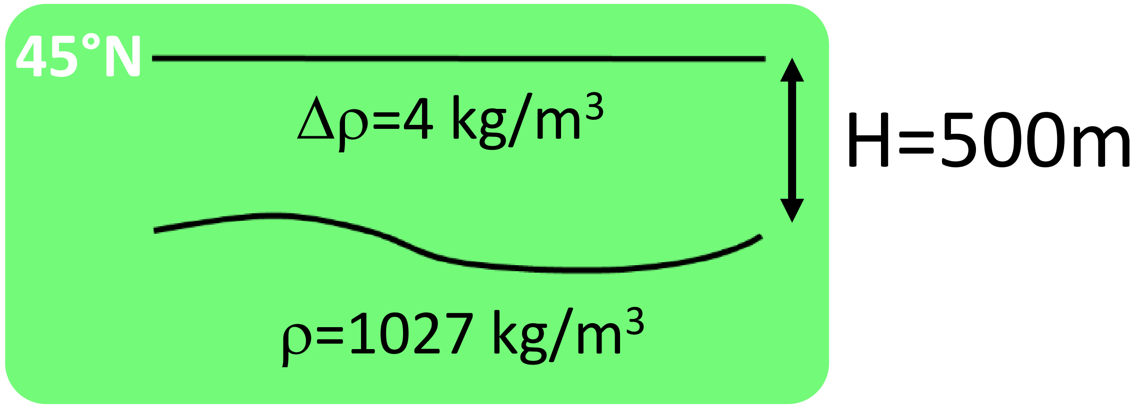
2) How long would it take to cross the Pacific between 130°W and 150°E ?

$$c_{WEST} = 6.37 \times 10^{-3}\, m.s^{-1}$$

- 45°N: $\mathrm{d} = 2\pi r cos\phi \times \frac{50°+30°}{360} = 28434 \times \frac{80}{360} = 6318\, km$
- $\mathrm{t} = \frac{d}{c_{WEST}} = 31\, years$



A baroclinic ocean Rossby wave



3) What is the fastest possible transit time for very large scale waves ?

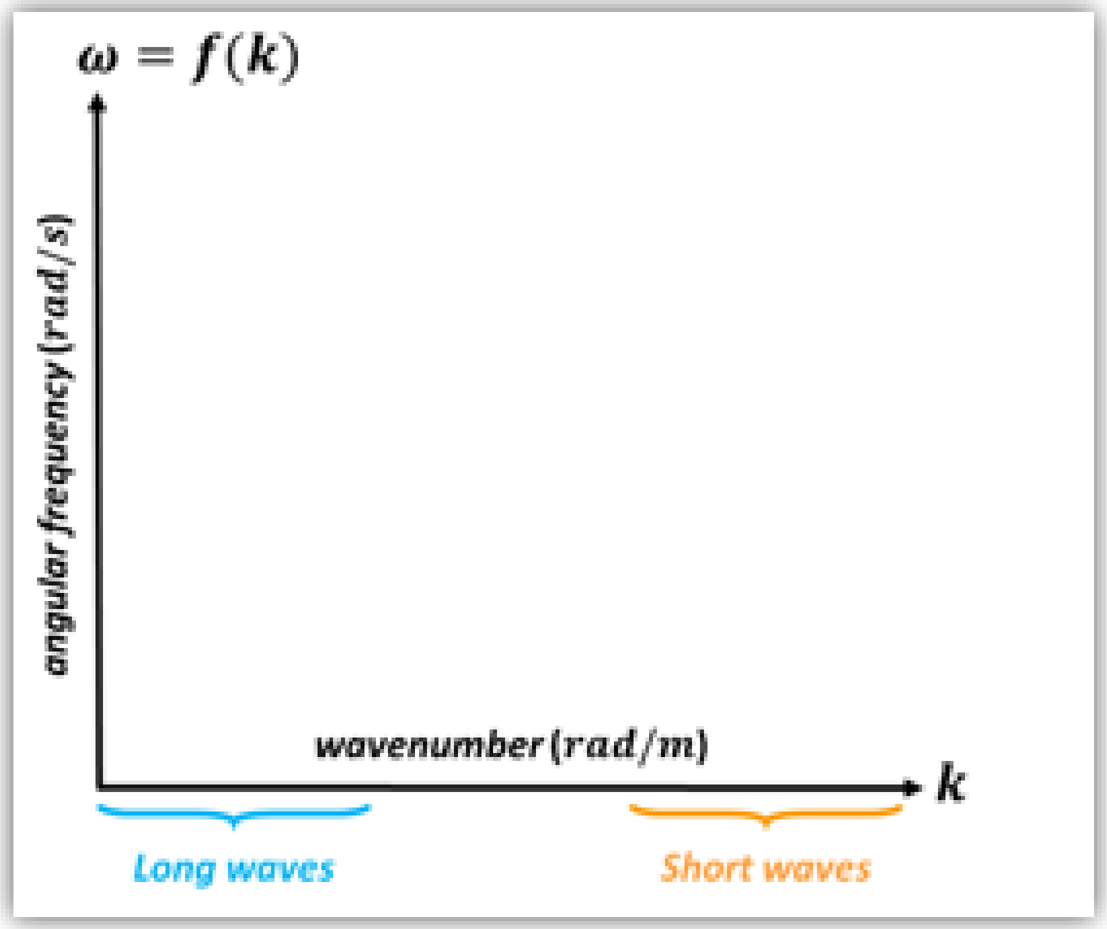
$$c_{WEST} = \frac{\beta}{l^2 + m^2 + L_R^{-2}}$$

- $d = 6318\ km$
- $l = 0, m = 0 \ \Rightarrow c_{max} = \frac{\beta}{L_R^{-2}}$
- $t_{min} = \frac{d}{c_{max}} = 6.8\ years$

Recap: Dispersion diagram

➪ The relationship between *ω* and *k* is called the dispersion relation.

➪ We represent the relationship between frequency and wavenumber ($\omega = f(k)$) on a diagram. This diagram is called a **dispersion diagram**.



- The horizontal axis is the wavenumber ($k = 2\pi/\lambda$), ranging from small wavenumbers (long waves) to large wavenumber (short waves).
- and the vertical axis is the frequency ($\omega = 2\pi/T$)

If we know the physical system, we know the relationship between ω and k. We can represent it as a curve on the dispersion diagram by associating a value of ω to each value of k. We can then work out the phase speed and the group speed for any wavelength.

On this graph:

⇢ **the phase speed** (c) is the arrow that points from the origin toward the curve (the ratio $\frac{\omega}{k}$)

⇢ **the group speed** (c_g) is the tangent to the curve ($\frac{\partial \omega}{\partial k}$)

Recap: Dispersion diagram

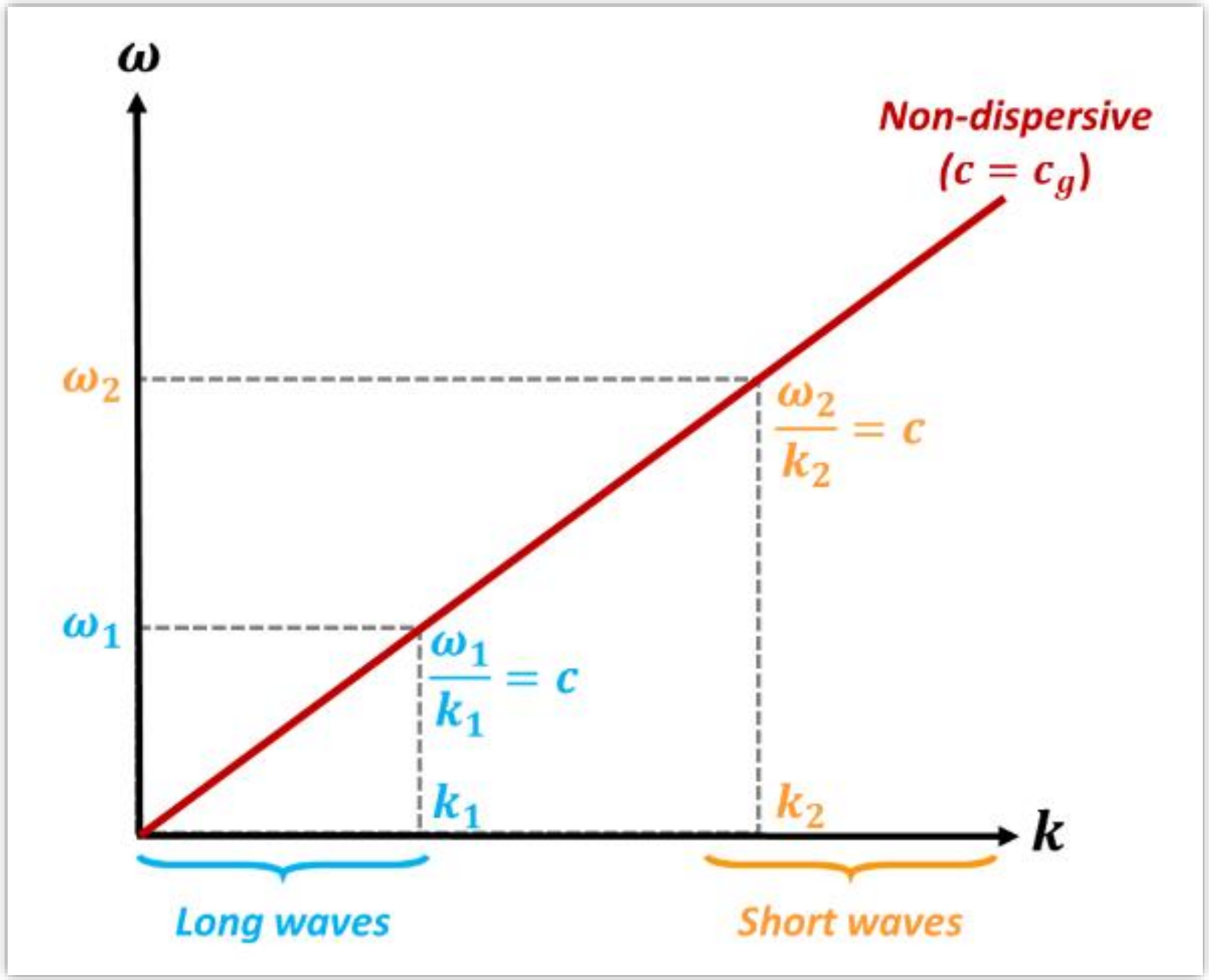
⇨ The relationship between *ω* and *k* is called the dispersion relation.

⇨ If this relationship is linear (and of course *ω=0* when *k=0*),

$$\frac{\partial \omega}{\partial k} = \frac{\omega}{k}, \quad c_g = c$$

↳ the wave is "non-dispersive"

For non-dispersive waves, all the wavelengths propagate at the same speed. A wave pattern (sum of different wavelength) will not change its shape during its propagation

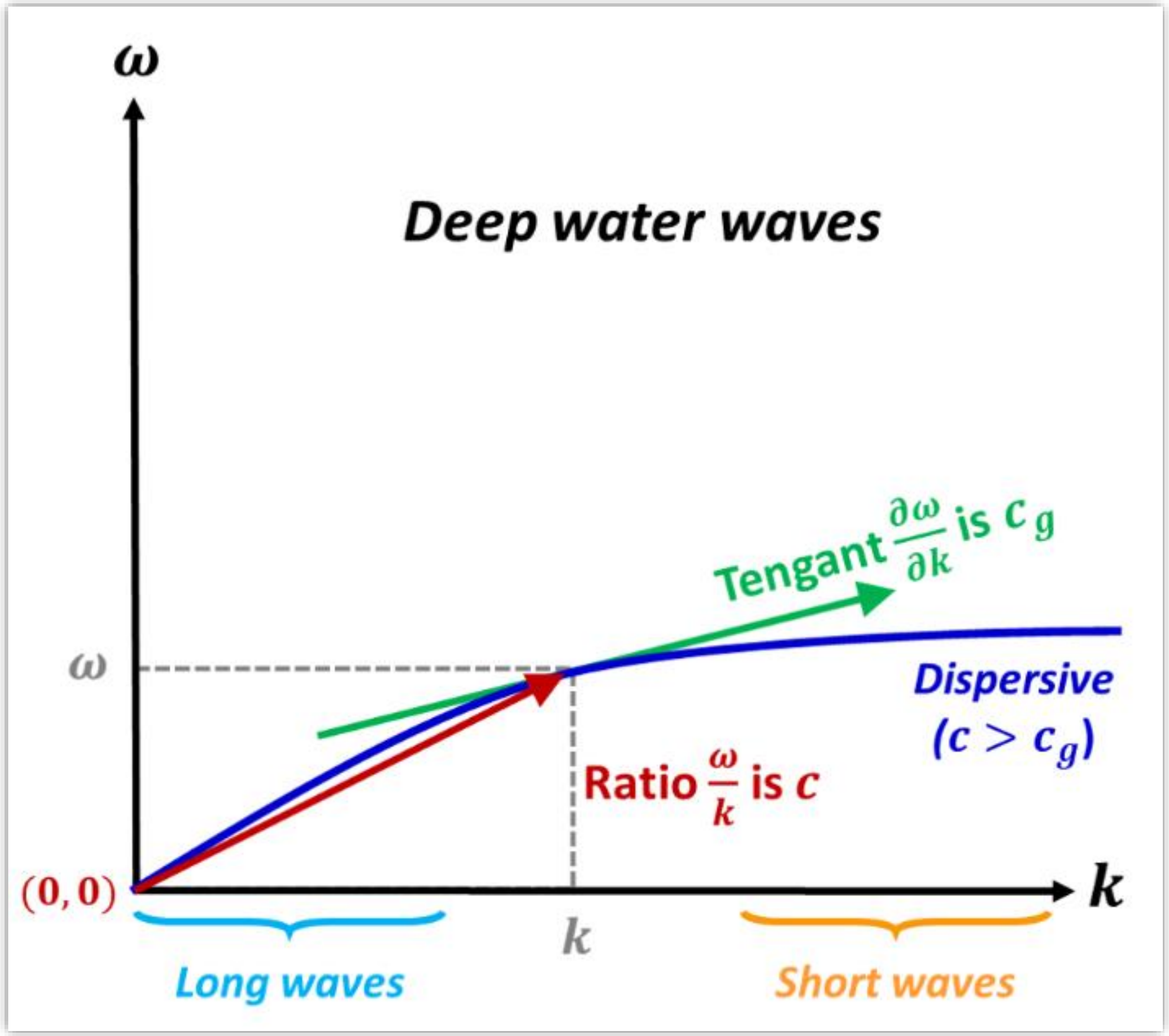


Recap: Dispersion diagram

⇨ The relationship between ω and k is called the dispersion relation.

⇢ **the phase speed** (c) is the arrow that points from the origin toward the curve (the ratio $\frac{\omega}{k}$)

⇢ **the group speed** (c_g) is the tangent to the curve ($\frac{\partial\omega}{\partial k}$)



Recap: Dispersion diagram

➩ The relationship between ω and k is called the dispersion relation.

• **Dispersive waves for which short waves go faster than long ones**: In such a physical system, the group speed is larger than the phase speed. So, the group packets propagate faster than the individual waves.

It is quite an unusual behaviour. This is the case of capillary waves, the tiny ripples on the surface of the water, for which the restoring forces is the surface tension.

This dynamics is outside our scope of interest for this course.