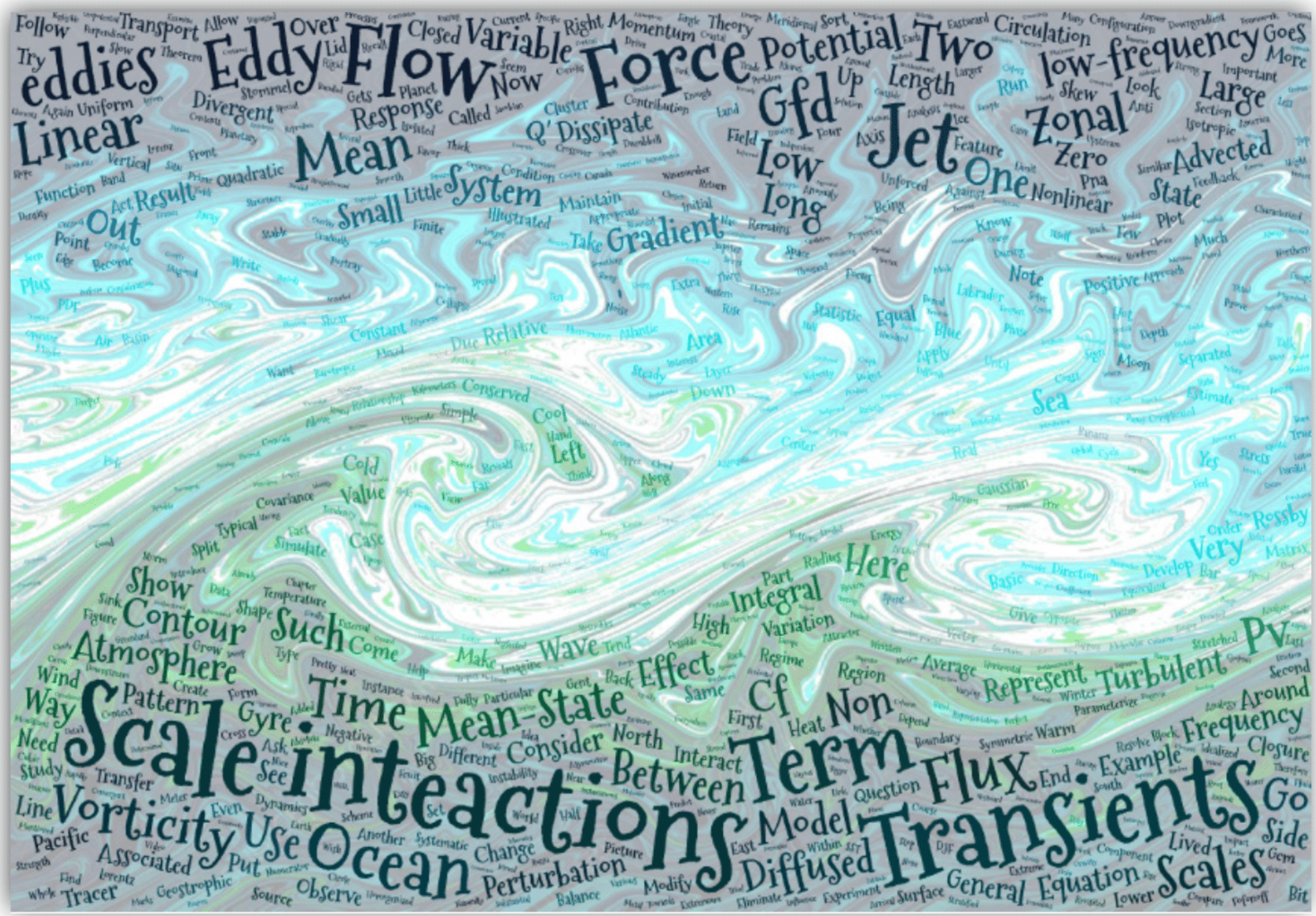
CHAPER 5
Scale interactions
in the Atmosphere and Ocean



CHAPTER 5
Scale interactions
in the Atmosphere and Ocean

GFD5 Contents

⇨ So far, we have focused on **linear dynamics**. We considered **perturbations** to the flow and these perturbations remained **small**, so quadratic terms in the perturbation could be neglected. This is equivalent to making a separation between a basic flow and the perturbation. In **#GFD3.3** and **#GFD3.4**, we discussed **instabilities** and found conditions under which these perturbations can grow exponentially. We are left wondering ***what happens when the perturbations become big enough*** that they can no longer be considered small relative to the magnitude of the background flow or its gradients? How does the ***perturbation interact with the mean flow***?

↳ So, in this chapter, we will introduce the idea of **scale interactions** in the atmosphere and ocean. We will see ***how these transient systems can modify or interact with the mean flow***.

▪ We will see how transient systems can be involved in ***large-scale forcing and transport***, i.e. how perturbations can transport properties and contribute to lower-frequency heat and momentum fluxes and potential vorticity (see **#GFD5.1**).

▪ As we cannot represent every single little transient system, we will look for a systematic way of representing their ***aggregate effect***, their statistical effect, on the average flow (see **#GFD5.2**). Can their effects be represented in terms of lower-frequency variations or average flow? This is ***closure***. One simple approach to closure is to consider these transient systems as a form of ***diffusion***. Barotropic or baroclinic gradients in the mean flow can create instabilities (see **#GFD3.3** and **#GFD3.4**), and these transient systems can in return eliminate the gradients by diffusion.

▪ In this way, transient systems can modify ***large-scale potential vorticity***. We will look at some examples in which transient systems affect large scale Ocean circulation (see **#GFD5.3**).

▪ We will also review some ***atmospheric examples*** of how transients interact with long-lived features to influence low-frequency variability (see **#GFD5.4**).

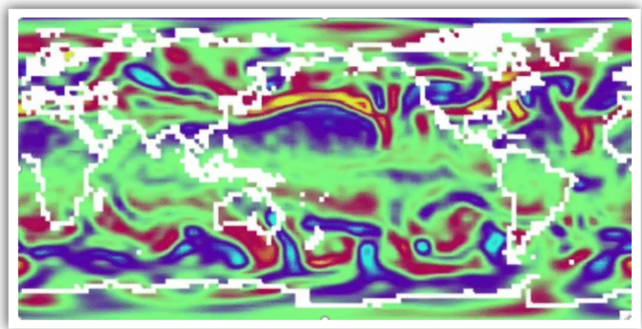
▪ Then, we will study the ***atmospheric response*** to other types of forcing anomalies. For example, we will see how the atmosphere responds to a change in the sea surface temperature, and how this basic response might be modified by the response of the transients (see **#GFD5.4b**).

▪ Finally, we will see how the ***flow on rotating planets tend to organize itself into zonal Jets*** (see **#GFD5.5**).

This chapter will also serve as an introduction to turbulent dynamics.

GFD5.1: Scale Interactions and Transient Forcing

5.1.a) Atmospheric illustration: 250mb relative vorticity



⇨ The video shows the **relative vorticity** in the atmosphere at 250 mb (from ECMWF ERA-Interim reanalysis) during boreal winter time (DJF).

↳ The patterns are very turbulent, portraying eddies propagating eastward in the extra-tropics.

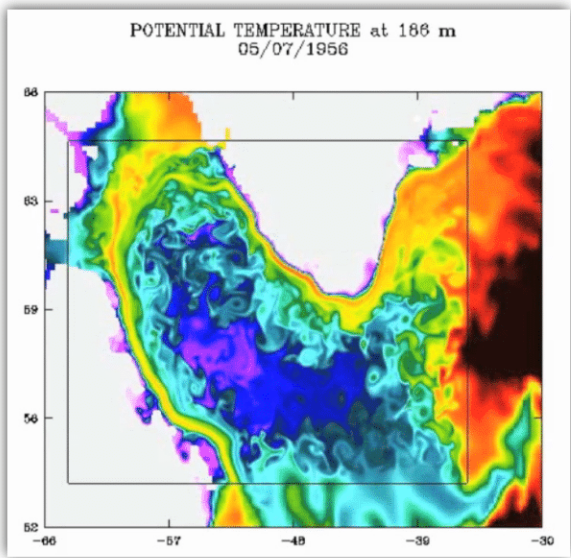
⇨ If you stare at these transient systems for long enough you can pick out some features:

• The variability is more active over the ***oceans*** than over the land and the northern Atlantic and Pacific Ocean basins are ***storm-track regions*** (see **#GFD1.1e**).

• Focusing on the western Pacific variability - the upstream part of the Pacific jet – we observe that features appear to be stretched out in the ***zonal direction***, while towards the east, turbulent patterns seem to be stretched out more in the ***meridional direction***.

↳ This is pretty systematic and as a result, ***there are consequences for how these transient systems interact with the Jet*** which they are traveling on (see **#GFD5.1d**).

5.1.b) Ocean illustration: Re-stratification of the Labrador Sea



⇨ Here is an example of ***heat transfer by transient systems influencing the mean state***.

The figure shows the ***potential temperature*** in the Labrador Sea at 165-meter depth from a model simulation, between Greenland and Canada.

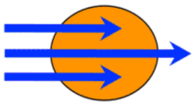
↳ We see a ***very strong annual cycle***. A ***sudden cooling*** at the surface (blue) marks the arrival of winter. It is associated with cold winds coming off Labrador. This ***cooling is convectively unstable and it is therefore mixed in the vertical*** very rapidly. As a result, the whole water column is cooled down to the bottom of the ocean.

⇨ ***After the winter, how is the stratified state reestablished?***

↳ ***Heat is transferred by turbulent eddies***. The "warm(er)" coastal current flowing around the Labrador Sea, along the Greenland and Canada coasts, is associated with strong gradients between the coast and the center of the basin. They favor the development of ***geostrophic eddies*** which transfer heat into the center of the basin and gradually re-establish the stratified state for the following summer.

5.1.c) Example: Momentum transport in zonal jets

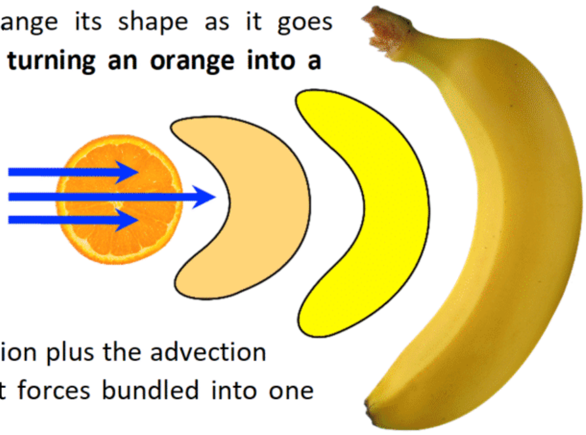
Here is ***an example in which transient eddy feedback maintains the mean flow***. Let's think about zonal jets and momentum transport in zonal jets.



⇨ We put a **cyclonic eddy in a zonal jet**. The figure shows a typical eddy (a closed contour) advected by a **sheared zonal jet**, maximum at the center.

What will be the effect of the jet on the shape of this eddy?

↳ It will shear it out. It will gradually change its shape as it goes downstream. This inspires a fresh fruit analogy, i.e. **turning an orange into a banana**.



⇨ And the fact that the eddy ends up looking like a banana is important for the general circulation. The variations of the jet follow this equation:

$$u_t + uu_x + vu_y = f(v - v_g) - \mathcal{D}$$

↳ There is a balance between the time variation plus the advection terms, and $f(v - v_g)$ (Coriolis and pressure gradient forces bundled into one term using the geostrophic wind) plus the dissipation.

⇨ Taking the time average of this equation shows that **the jet is diffused by dissipation and powered by the momentum fluxes**, i.e. mean dissipation is balanced by the quadratic advection terms:

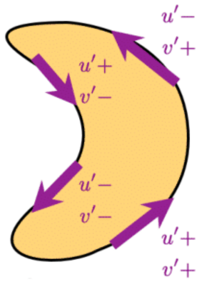
$$\overline{u'u'_x} + \overline{v'u'_y} = -\mathcal{D}$$

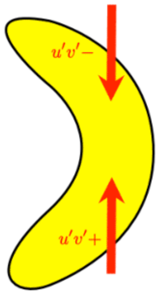
↳ The fluxes of momentum can be reformulated as the divergence of a flux, i.e.: $(\overline{u'u'})_x + (\overline{u'v'})_y = -\mathcal{D} + \overline{u'd'}$. The term $(\overline{u'v'})_y$ - a covariance between u' and v' - is the most important term in this equation.

⇨ We can estimate its contribution by looking at the flow as it goes around one of these banana-shaped eddies.

• In the northern half of this eddy, the perturbation flow is going northwestwards so there is a negative covariance between u' and v'. On the way back, u' is positive while v' is negative, i.e. a negative covariance between u' and v'.

• In the southern half of the eddy, the flow is either southwestward or northeastwards, with a positive covariance between u' and v'.





↳ In the north, we observe a <u>southward flux of eastward momentum</u>, while in the south there is a <u>northward flux of eastward momentum</u>. This gives rise to a ***convergence of momentum*** flux which will ***accelerate the jet towards the east*** and help to ***maintain the jet against the dissipation*** term on the RHS. This is how the jet is maintained by mature finite amplitude synoptic systems.

An eddy that gets stretched out and deformed by a jet, will produce a convergent momentum flux that maintains the jet against dissipation.

5.1.d) General consideration for tracer transport

⇨ Let's formalize this by considering this generic non-linear system. The **tendency equation** with advection and forcing of a tracer q (potential vorticity for instance) can be written:

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{F} - \mathcal{D}$$

↳ $\frac{\partial q}{\partial t}$ plus the advection term equal sources and sinks, i.e. forcing and dissipation.

• The forcing could be the wind stress for instance and dissipative sink could be diffusion.

• The advection term can be written as a Jacobian of ψ and q ($J(\psi, q)$) (see **#GFD2.3h**):

$$\frac{\partial q}{\partial t} + J(\psi, q) = \mathcal{F} - \mathcal{D}$$

$J(\psi, q)$ is the advection of q by this non-divergent flow associated with the stream function ψ, such that $u = -\frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$

⇨ We now split up the flow (of which the potential vorticity q is a diagnostic) into two components, the **average flow** (noted with a bar, $\bar{q}$ and $\bar{\psi}$) and the **perturbation flow** which is varying in time (noted with a prime): $q = \bar{q} + q'$ and $\psi = \bar{\psi} + \psi'$. The tendency equation can be written as follows:

By definition $\frac{d\bar{q}}{dt} = 0$

$$\frac{\partial q'}{\partial t} + \underbrace{J(\bar{\psi}, \bar{q})}_{\text{mean flow advection}} + \underbrace{J(\bar{\psi}, q') + J(\psi', \bar{q})}_{\text{linear waves}} + \underbrace{J(\psi', q')}_{\text{turbulence}} = \mathcal{F} - \mathcal{D}$$

↳ The non-linear quadratic advection term is split into four terms:

1) ***mean flow advection***: the mean potential vorticity being transported by the mean flow.
2) ***2 linear terms***: perturbation PV being transported by the mean flow and the perturbation flow transporting the mean PV. These terms gave us waves and instabilities.
3) a quadratic term $J(\psi', q')$.

↳ In chapters 2, 3 and 4, we neglected the contribution of the quadratic term because it is quadratic in the perturbation and the perturbation was small. If the perturbation is not small anymore, this term is not negligible and we need to study what this term does.

⇨ If we are interested in the **systematic effect of this quadratic term**, its non-zero time mean, we can form the time mean of the tendency equation, leading to a ***budget equation*** for q:

$$J(\overline{\psi},\overline{q}) = -\overline{J(\psi',q')} + \overline{\mathcal{F}} - \overline{\mathcal{D}}$$

transient "forcing"

↳ The advection of the mean tracer by the mean flow will be balanced by the mean of the forcing, the mean of the dissipation, and the mean of this transient forcing term. So, we now put this term on the RHS of the budget equation and consider it as a forcing term, a forcing by the transient fluxes. We discussed this in **#GFD1.1e**.

⇨ Just in passing, we note that for the special case of time-independent unforced flow (no time variation and no forcing/dissipation) there is ***a time-independent conservation law*** so that advection is equal to zero:

$$J(\psi, q) = 0$$

↳ q would be strictly a function of ψ ($q = q(\psi)$) meaning that contours of q would overlay contours of ψ. This describes a closed circulation - q contours coincide with ψ contours. We will come back to this no-advection state in which nonlinearity is associated with closed circulations in **#GFD5.3c**.

↳ The figure shows some turbulence. Closed contours for which $J(\psi, q) = 0$ can be considered either for little turbulent eddies or for something much bigger like ocean gyres.

GFD5.2: Effect of Transients on the Mean Flow: Closure & Diffusion

5.2.a) Forcing due to transients: Closure

⇨ Imagine we wish to simulate or predict the slow, large-scale flow. Because the ***system is nonlinear*** the fast, small-scale component (maybe unresolved) will affect the slow, large scale variability.

↳ **Closure** is the systematic study of how we can represent the feedback of the **transients** on the lower-frequency flow variation.

⇨ Consider a non-linear development of a zonal wind u according to the following abstract non-linear equation:

$$\frac{du}{dt} + uu + ru = 0$$

↳The term uu is quadratic and ru is linear. It is an idealized generic equation.

⇨ Let's examine this equation in term of low-frequency variations by taking the time average or the low-frequency component:

$$\frac{d\overline{u}}{dt} + \overline{uu} + r\overline{u} = 0$$

☝ ***We want to solve this equation for*** $\overline{u}$. The problem is that we don't know $\overline{uu}$:

$$\overline{uu} \neq \overline{u}\,\overline{u} \quad \text{, it is} \quad \overline{uu} = \overline{u}\,\overline{u} + \overline{u'u'}$$

↳ In $\overline{uu}$, there is the contribution of the transients that need to be addressed.

⇨ If we try to write an equation for the quadratic term $\overline{uu}$ (by multiplying the non-linear abstract equation by u), we end up with an equation in which there is a cubic term $\overline{uuu}$, which is of no help:

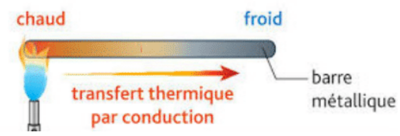
$$\frac{1}{2}\frac{d}{dt}\overline{uu} + \overline{uuu} + r\overline{uu} = 0$$

⇨ We can do it again as many times as we want but at some point, we will need to represent the $(n + 1)^{\text{th}}$-order term in terms of the n^{th} order term.

↳ To keep it simple, ***we will need to represent the quadratic term $\overline{u'u'}$ in terms of the mean flow***. And to do so, we must make <u>additional physical assumptions</u>. This is <u>**turbulent closure**</u>.

5.2.b) Diffusion and diffusivity

⇨ There are various approaches to closure and one we have already mentioned is ***diffusion***. We can use diffusion to represent the systematic effect of transients in terms of the mean flow. We make the <u>**analogy**</u> that the effect of the transients is similar to ***molecular diffusion***.



↳ In a ***metal bar*** which is hot at one end and cold at the other, molecular diffusion will transport the heat from the hot end to the cold end and gradually the temperature will become uniform. This is because heat is transported downgradient.

↳ Here, ***we assume that geostrophic eddies act in a similar way***. If there is a gradient in some larger-scale field, the geostrophic eddies will tend to smooth out this gradient.

⇨ Let's go back to our tracer equation and consider a diffusive representation for the flux of the tracer q. For the moment, we ignore other forms of forcing and dissipation. We consider advection by a non-divergent flow:

$$\frac{\partial q}{\partial t} + \nabla.\mathbf{v}q = 0$$

↳ We split this advection term into the advection by the time mean and the transient eddy's:

$$\frac{\partial \overline{q}}{\partial t} + \nabla.\overline{\mathbf{v}}\,\overline{q} = -\nabla.\overline{\mathbf{v}'q'}$$

⇨ Let's represent the eddy covariance term through analogy with molecular diffusion, i.e. transport down the mean gradient, so:

$$\overline{\mathbf{v}'q'} = -K\nabla\overline{q}$$

↳ This way, the transient forcing term will transfer properties down gradient and will smooth out gradients. We can substitute it into the non-divergent flow equation in which the substantial derivative of $\overline{q}$ is represented in term of $\overline{q}$, as:

$$\frac{D\overline{q}}{Dt} = \nabla.(K\nabla\overline{q}) \quad (= \nabla.F)$$

This is our assumption for how the transients are going to impact the mean flow.
It is a parameterization/closure

⇨ In general, K is a matrix, a ***second rank tensor***. Diffusion is usually not <u>***isotropic***</u> for large-scale flows, meaning that diffusion in some directions might be stronger than in other directions. For example, the flux $\overline{v'q'}$ is represented by a coefficient $(-\kappa^{vy})$ times the meridional gradient and another coefficient $(-\kappa^{vz})$ for the vertical gradient:

$$\overline{\mathbf{v}'q'} = -\kappa^{vy}\frac{\partial \overline{q}}{\partial y} - \kappa^{vz}\frac{\partial \overline{q}}{\partial z}$$

↳ These coefficients come from turbulence theory. We can estimate them by using some scaling arguments: $\kappa^{vy} \sim v'l'$ where v' is a typical eddy velocity and l' is a "mixing length".

5.2.c) Symmetric and asymmetric diffusion

⇨ We can decompose $\boldsymbol{K}$ into symmetric and antisymmetric parts $\boldsymbol{K} = \boldsymbol{S} + \boldsymbol{A}$.

• The simplest specification is ***isotropic downgradient diffusion***.

$$K = S = \begin{pmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{pmatrix}$$

↳ The 3×3 $\boldsymbol{K}$ matrix is just a diagonal matrix with the same constant down the diagonal and the diffusive flux is downgradient:

$$F = -\kappa \nabla \overline{q}$$

• In general, a downgradient flux is associated with symmetric matrices $\boldsymbol{S}$.

⇨ But this matrix can have an anti-symmetric part $\boldsymbol{A}$, such that: $F = -A\nabla q$, and:

$$F.\nabla \overline{q} = -(A\nabla \overline{q}).\nabla \overline{q} = 0$$

↳ $\boldsymbol{A}$ will consist of off-diagonal elements of opposite sign. As a result, the result of this matrix multiplied by a vector will be perpendicular to that vector ($\boldsymbol{A}x \perp x$). This means that this diffusive flux is neither upgradient nor downgradient, but it is parallel to the contours of the mean state. This flux is called a “skew flux”.

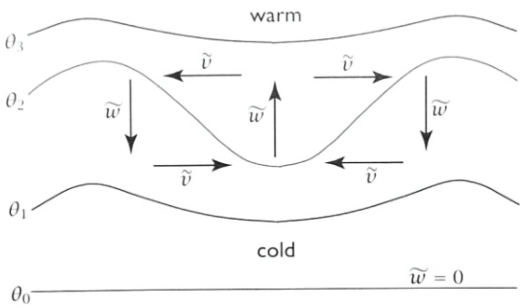
A **skew flux** is thus equivalent to ***advection*** by a non-divergent flow. Its velocity can be represented by a stream function: $\tilde{\mathbf{v}} = \nabla_{\wedge}\psi$. Therefore, ***the skew flux does not change the gradient, it goes along the gradient***.

⇨ *Whether or not it is appropriate to use straightforward isotropic downgradient diffusion or have some anti-symmetric terms in the matrix depends on the time-scale we analyze and on the tracer variable (conserved or not) for which we are trying to represent the effect of* **transients**.

5.2.d) Parameterization

⇨ Here is an example of a parameterization that is often used in ocean models.

📖 It is more difficult to model the ocean than the atmosphere because the Rossby radius is significantly smaller, a few hundred kilometers vs. a thousand kilometers. Most atmospheric models now have no trouble resolving these scales. To resolve geostrophic **eddies** in the ocean, one needs to use a substantially higher resolution which is quite expensive to run on a computer, especially for long simulations. There is a trade-off between the length of the simulation and how much you can resolve.



↳ Imagine that the ***geostrophic eddy-scales are not fully resolved*** in a chosen model configuration. We thus have to represent their effect on the larger scales in some other way. The *Gent and McWilliams* parameterization is one approach to doing this. It is illustrated in the figure showing density surfaces near the thermocline.

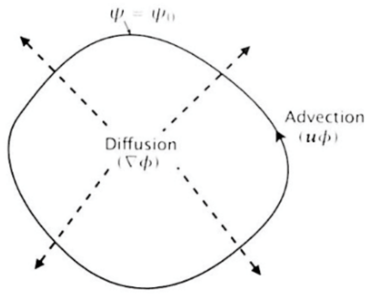
▪ Isotropic diffusion would simply flatten the density gradients in the vertical.

▪ Another way to do this, in agreement with the ***mechanism of baroclinic instability*** we studied in **#GFD3.4**, is to flatten-out the tilted density contours by ***advective flow*** causing a transfer of energy between the potential energy stored in the slope of isentropes and kinetic energy of the growing systems. The *Gent and McWilliams* scheme formulates the eddy closure in terms of asymmetric diffusion of thickness, i.e. the circulation associated with this scheme is represented by a skew flux in the diffusion scheme.

GFD5.3: Systematic Effects of Transient Eddies on Ocean Gyres

5.3.a) Potential vorticity homogenization

⇨ We discuss now ***the systematic effects of eddies on ocean gyres***. We study **a simple case** in which the effect of transients is represented as **an isotropic diffusion** (see **#GFD5.2b**) and we will discuss the diffusion of potential vorticity (PV).



↳ We are going to look at very large-scales in the Ocean and ask ourselves what is the effect of adding some diffusion of PV on the mean-field of PV? It is useful to study potential vorticity in this context for two reasons.

1) Because PV is conserved following the motion, so it is meaningful to talk about its diffusion.

2) Knowing the potential vorticity implies knowing the flow. There is an intimate connection between the large-scale flow and the large-scale potential vorticity.

⇨ Here is the generic advection-forcing-dissipation equation for the conservation of the potential vorticity:

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \nabla.(\kappa \nabla q) + \mathcal{S}$$

- The effect of transients on PV are represented as an isotropic diffusion (see **#GFD5.2b**)
- ***S*** is a source of potential vorticity

↳ On the large-scales, we consider a model of **steady non-divergent** flow, **isolated from any sources** of PV.

- The flow being <u>steady</u> means that time variations can be crossed-out.
- We are in a region that is <u>sheltered from wind stress forcing</u>, i.e. away from the surface of the Ocean - in the deeper ocean where the flow does not feel the forcing effect ***S***.
- <u>Non-divergent</u> flow implies that $\boldsymbol{v}.\nabla q = \nabla.(\boldsymbol{v}q)$

↳ The PV conservation is written $\nabla.(\mathbf{v}q) = \nabla.(\kappa \nabla q)$

⇨ Let's consider a **<u>closed contour</u>** of the flow and **<u>estimate the integral</u>** of this equality over the area delimited by this contour.

$$\iint_A \nabla.(\mathbf{v}q)\, dA = \iint_A \nabla.(\kappa \nabla q)\, dA$$

• <u>The left-hand side</u> integrates to zero.

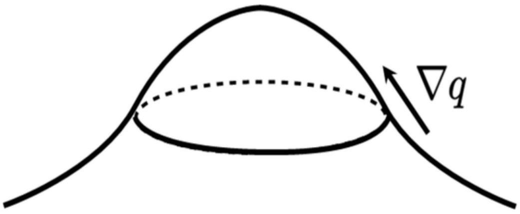
$$\iint_A \nabla.(\mathbf{v}q)\, dA = \oint (\mathbf{v}q).\hat{\mathbf{n}}\, dl = q \oint \mathbf{v}.\hat{\mathbf{n}}\, dl = q \iint_A \nabla.\mathbf{v}\, dA = 0$$

1) Following the divergence theorem (see **#GFD1.3a**), the area integral of a divergence is the line integral around that contour of the flux of q perpendicular to the contour.
2) Since q is constant on this contour (unforced flow) then q can come out of the integral which is now the line integral of the flow perpendicular to the contour.
3) Following the divergence theorem again, the line integral can be rewritten in terms of the area integral of the divergence of the flow.
4) Since we imposed the flow to be non-divergent then the LHS is equal to zero within a flow-contour (which since we have steady free flow is also a q-contour).

• <u>The right-hand side</u> must also be zero within the area.
→ Using the divergence theorem, the area integral of the divergence of $\kappa\nabla q$ must be equal to κ times the line integral of the component of gradient of q that is perpendicular to the boundary.

$$\iint_A \nabla.(\kappa \nabla q)\, dA = \oint \kappa \nabla q.\hat{\mathbf{n}}\, dl = 0$$

↳ This result means that as we integrate around this flow contour, the gradient perpendicular to this contour must integrate to zero in a steady unforced flow.



As illustrated on the schematic, **this cannot be true if the contour encloses an extremum of q**. In this case, the gradient of q perpendicular to a flow-contour is not going to integrate to zero. This means that the eddies are going to transfer properties to the mean flow until such a point is that it does become zero. The extremum in q is going to get eroded and eliminated, until a state is achieved in which the potential vorticity is uniform (one constant value) throughout the region

Talking of an extremum in potential vorticity reminds us of the Rayleigh criterion for barotropic instability (see **#GFD3.3e**). An extremum in PV is a necessary condition to create instabilities and generate transient flow. In turn, the transient flow will act to eliminate the source of the instability. The final result is that all gradients of potential vorticity will be eradicated, resulting in the **homogenization of the potential vorticity** to a uniform value (in regions remote from sources of q).

5.3.b) Examples in models and observations

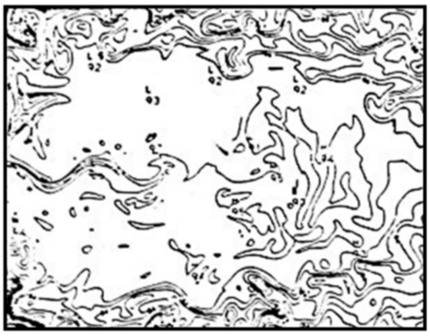
⇨ Here are a couple of examples.

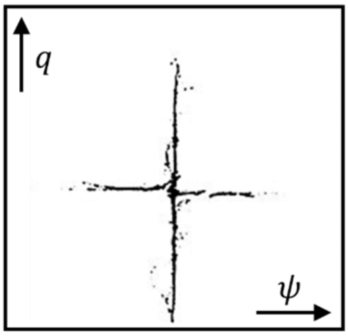
• On the right is the potential vorticity from an ocean model. It is not the top layer of the ocean. It is at a depth where the flow is isolated from forcing. On the top panel, we observe a large region of uniform potential vorticity (no horizontal gradients) where the gyre is active. The gradients are pushed out to the edge, where there is no flow.

This is illustrated on the bottom panel, in which values of q (vertical axis) are plotted as a function of ψ (horizontal axis). It comes down to this ultimate state where

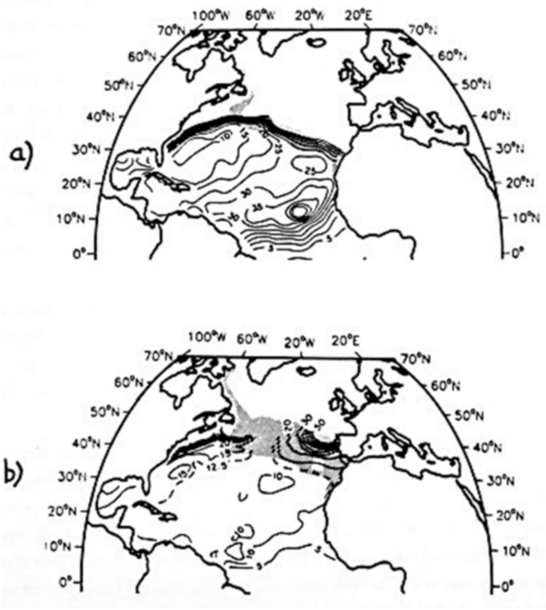
1) either there are variations in q but in that case $\psi = 0$, i.e. there is no flow, i.e. the β-effect outside the flow region,
2) or ψ is varying (there is a mean-flow), in which case q is uniform (within the gyre).

QG model - mid-level PV





Observed PV on isopycnal surfaces



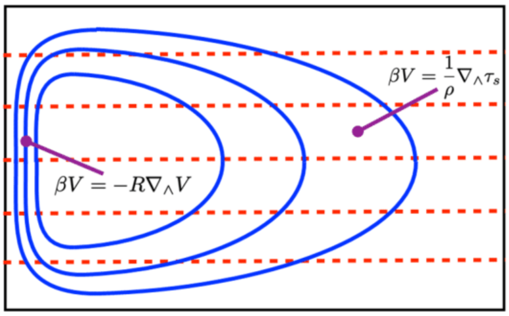
• On the right is an example of ocean gyres from observations. The lower figure shows a deeper layer and uniform values of potential vorticity can be observed within the gyre.

↳ The upper figure shows a layer nearer the surface where the flow is not isolated from the surface forcing. q values are not uniform but portray closed contours around the gyre. This is different from the classical large-scale ocean circulation theories (see **#GFD5.3c**).

5.3.c) Stommel vs. Fofonoff

⇨ In this section, we focus on the large-scale ocean circulation and we contrast two paradigms of large-scale ocean circulation theory.

• **The first one is the Stommel solution**. The Sverdrup term, i.e. the advection of planetary vorticity (βV), is balanced by forcing and friction, so:



$$\beta V = \frac{1}{\rho}\nabla_\wedge \tau_s - R\nabla_\wedge V$$

Contours of potential vorticity are parallel to latitude lines.

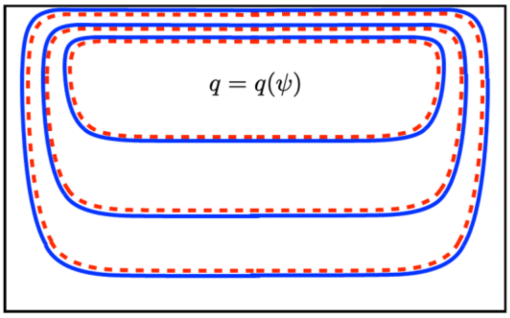
➢ As the flow goes south, the wind stress forces it to cross these contours. The flow is forced to change its potential vorticity.

➢ As the flow goes back north, it has to become an intense jet, so the friction term can be large enough to remove the vorticity that was injected by the wind stress.

↳ The **Stommel gyre is forced and dissipated** and the absolute vorticity is always changing.

• **There is another solution in an unforced context**.

➢ Imagine an unforced system, an Ocean gyre in which the **potential vorticity is conserved** – it never changes. This means that the flow goes around a flow-contour, the PV-contour remains parallel to that flow contour. An **unforced barotropic** system follows:



$$\frac{dq}{dt} = \frac{\partial q}{\partial t} + \boldsymbol{v}\nabla q = 0 \quad \text{with} \quad q = \xi + f$$

$$v.\nabla\xi + \beta v = v.\nabla q = 0$$

➢ There is a **cancellation between the advection of relative vorticity and the advection of planetary vorticity**. The sum of the two is conserved, i.e. PV is conserved and $J(\psi, q) = 0$ (see **#GFD5.1c**). This means that q is strictly a function of the stream function ($q = q(\psi)$). In this barotropic case, the potential vorticity is the relative vorticity plus βy and is a function of ψ, so that:

$$\nabla^2\psi + \beta y = fn(\psi)$$

We do not know this function

↳ This is a kind of opposite extreme view of the ocean circulation compared to the forced dissipative Stommel gyre. It is called a **Fofonoff gyre**. We imagine that the ocean circulation gets into this state due to the action of transient eddies modifying the potential vorticity field.

5.3.d) Diffusion and the strength of the gyre

⇨ We do not know the relationship between q and ψ. The simplest relationship we can consider is a linear relationship, meaning that the gradient of q is proportional to the gradient of ψ:

$$\nabla q \approx \frac{dq}{d\psi}\nabla\psi$$

⇨ The budget of (steady) q in the upper layer involves some forcing and dissipation:

$$J(\psi, q) = \nabla.(\kappa\nabla q) + \mathcal{S}$$

⇨ To estimate the value of the linear coefficient, we can integrate this equation within a streamline ψ, i.e. around a closed contour of q.

↳ For a ***non-divergent*** flow, the LHS is zero (see **#GFD5.3a**), which reveals a balance between a dissipative term (from the transients) and a forcing term:

$$0 = \iint_A \nabla.(\kappa\nabla q)\, dA + \iint_A \mathcal{S}\, dA$$

⇨ Using the divergence theorem (see **#GFD1.3a**), we can eliminate the divergence of the diffusion by transforming the area integral into the line integral of $\kappa\nabla q$. ∇q is expressed in terms of $\nabla\psi$ using the linear relation we hypothesized. It follows:

$$\Rightarrow \quad \iint_A \mathcal{S}\, dA = -\oint_\psi \kappa\nabla q.\hat{\mathbf{n}}\, dl = -\oint_\psi \kappa\frac{dq}{d\psi}\nabla\psi.\hat{\mathbf{n}}\, dl$$

↳ As $\frac{dq}{d\psi}$ is a constant, it can be moved outside the integral and we get: $\frac{dq}{d\psi} = -\frac{\iint_A \mathcal{S}\, dA}{\oint_\psi \kappa\, \mathbf{v}.dl}$

▪ The linear relationship between q and ψ is determined by integrals of forcing and dissipation around the closed gyre circulation.

▪ Integrated eddy diffusion provides the link between the q / ψ relationship and **the strength of the circulation**.

▪ In regions isolated from forcing, the numerator is zero but the denominator is non-zero, so the field of q must be uniform. q is homogenized as seen in **#GFD5.3a**.

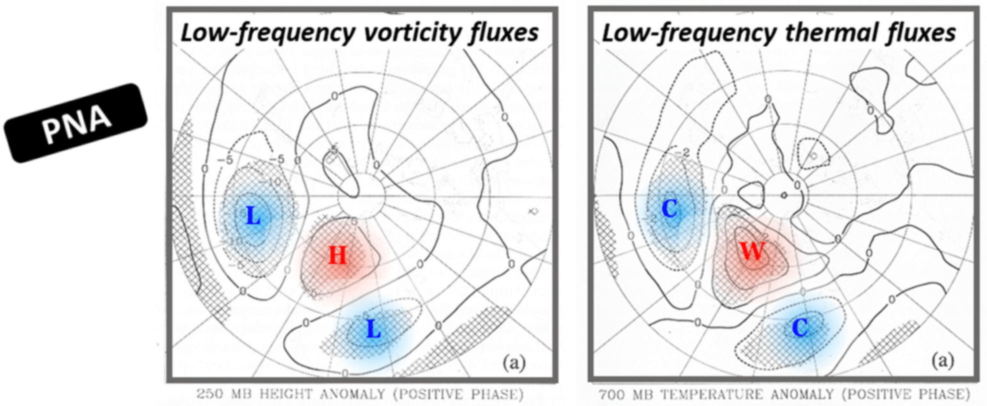
↳ This is the other extreme view of the ocean circulation.

GFD5.4: Examples of Scale Interactions in the Atmosphere

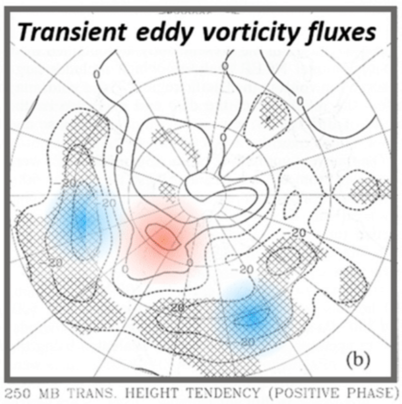
5.4.a) Long-lived atmospheric flow anomalies

⇨ In this section, we show examples from the atmosphere and focus on the maintenance of low-frequency variability. We ask the questions:

- How does the atmosphere stay in a particular configuration over long periods of time?
- What is the relationship between low-frequency variations and the fast-transient eddies?



⇨ Here is a **first example** (from Sheng et al., 1998) of a very important feature of the low-frequency variability of the Atmosphere. The figure above shows the result of a composite analysis of the northern hemisphere (bottom is North America) emphasizing a typical Low-High-Low Cold-Warm-Cold configuration associated with the Pacific North American (PNA) pattern. The atmosphere very often finds itself in this pattern, either in its positive or negative ***phase***.

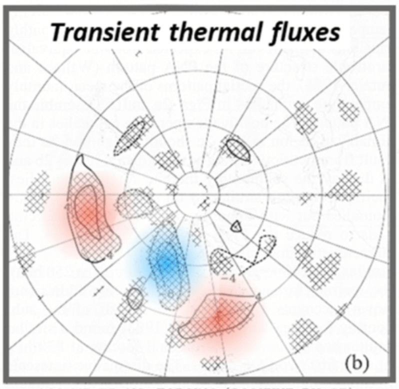


↳ <u>**The question here is**</u>: Does this pattern get ***dissipated*** by the transient eddies - their systematic effect - or is it ***reinforced***?

• On the left is the geopotential height tendency ***due to the transient eddy fluxes*** during these episodes of positive PNA. The pattern is in phase with the low-frequency pattern, i.e. Negative-Positive-Negative configuration, thus reinforcing the low-frequency pattern during episodes of positive PNA. The transient eddy momentum fluxes act to maintain the pattern in the geopotential height.

• **Conversely**, the transient fluxes of ***temperature*** show a Positive-Negative-Positive configuration, which tends to warm up cold regions and cool down where it is warm. Transient fluxes of temperature are thus dissipating the temperature signature.

⇨ In conclusion, observational analyses consistently show that high-frequency transient eddy vorticity fluxes reinforce the low-frequency patterns, while transient thermal fluxes dissipate them.

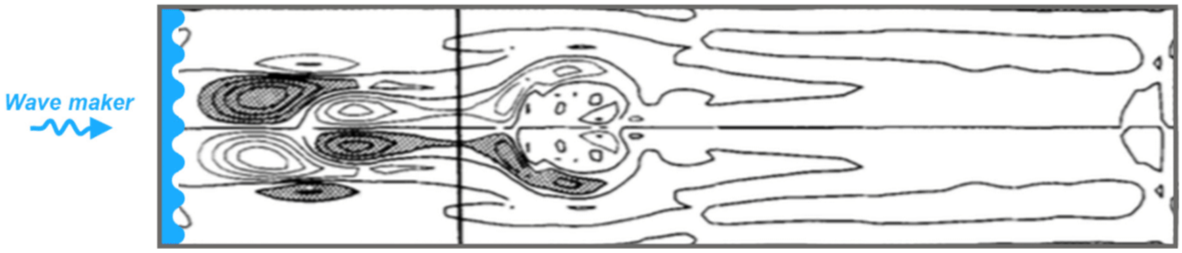


700 MB TRANS. FORCING (POSITIVE PHASE)

⇨ Here is a **second example** (from Haynes and Marshall, 1986) of a long-lived atmospheric feature called **blocking** that can be observed over Europe. It manifests as a High to the north and a Low to the south. In the wintertime, it brings very cold air from Russia to western Europe. This configuration remained there for a long time in February 2012.

↳ It is interesting to analyze what transient systems coming across the Atlantic do to this pattern: do they sweep it away or do they act to maintain it?

• Here are the results from an idealized model experiment in which a **wavemaker** is put upstream to generate high-frequency disturbances. The potential vorticity flux divergence shows that these transient eddies impinge upon this reversed dipole downstream. The transfer of potential vorticity is such as to maintain the stable block against dissipation.



⇨ In conclusion, there is evidence that high-frequency transient eddy vorticity fluxes maintain this blocking configuration and this explains why it is such a long-lived feature.

5.4.b) Transient feedback on a forced response

⇨ In this section, we study how transient eddies modify the atmospheric response to some other external forcing (from Hall et al., 2001). We recall the potential vorticity development equation:

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{F} - \mathcal{D}$$

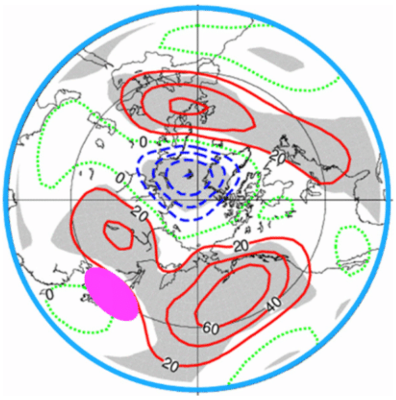
↳ The time average of the potential vorticity flux is the average forcing minus the average dissipation called $\mathcal{G}$:

$$\overline{\mathbf{v}.\nabla q} = \overline{\mathcal{F}} - \overline{\mathcal{D}} = \mathcal{G}$$

↳ We then split the time-averaged potential vorticity flux $\overline{\boldsymbol{v}.\nabla q}$ into two components: the flux by the time means ($\overline{\boldsymbol{v}}.\nabla\overline{q}$) and the transient term ($\overline{\boldsymbol{v}'.\nabla q'}$). The latter is put on the RHS to be considered as a forcing (as in **#GFD1.1e** and **#GFD5.1c**) and the sum of the forcing is then called $\mathcal{H}$:

$$\overline{\mathbf{v}}.\nabla\overline{q} = \overline{\mathcal{F}} - \overline{\mathcal{D}} - \overline{\mathbf{v}'.\nabla q'} = \mathcal{H}$$

⇨ Two forcings: one is the real forcing ($\mathcal{G}$), and another is a forcing that includes the transient eddy fluxes ($\mathcal{H}$). These two forcing terms can be used to drive a model.

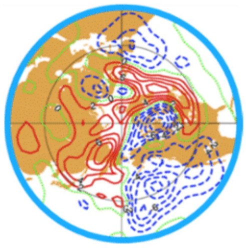


SET1 $\mathcal{G}$ is diagnosed from data and is used to drive a simple atmospheric General Circulation Model (GCM):

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{G}$$

↳ In a second experiment, a small perturbation f' is prescribed. Here, we add a perturbation to the sea surface temperature in the western Pacific. We run the GCM again with this extra bit of forcing associated with this perturbation:

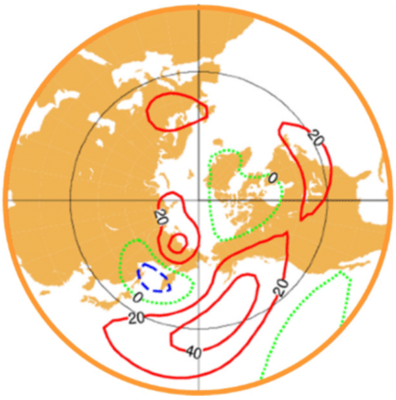
$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{G} + f'$$



⇨ The difference between the two long runs (see figure above) shows a ***global response***, characterized by a large high downstream in the Pacific, a low over the north pole, and another high over the Atlantic-European sector.

⇨ We ask now what the contribution of the transients in this response is. As these two experiments will not necessarily have the same value of the transient component to the forcing ($\overline{\boldsymbol{v}'.\nabla q'}$), we diagnose the difference in transient forcing between the two experiments ($\Delta(v'.\nabla q')$, see figure on the right).

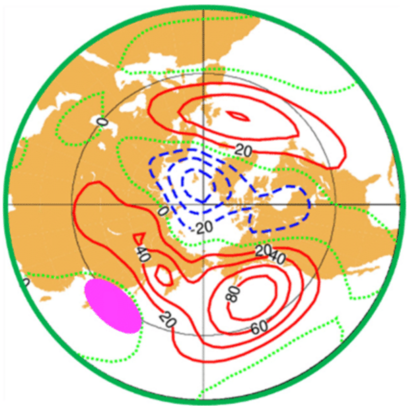
SET2 We now force the GCM with the forcing $\mathcal{H}$ (instead of $\mathcal{G}$) : $\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{H}$



↳ If we initialize the model with its time-mean $\overline{q}$, there is no development because $\mathcal{H}$ is what is needed to stop any development. Therefore, the initial conditions perpetuate.

Then, we add the same small perturbation f' to this forcing and perform another simulation. We have a model in which the transient part is already taken into account in the forcing and we apply a small perturbation. We have a linear perturbation model. In response to the Pacific SST anomaly, we get a response which is not as global as in **SET1** (which was the fully nonlinear response with modified transient eddy feedback). The linear response is basically just a Pacific response.

SET3 So then the question is: ***Is the difference between these two sets of experiments due to the change in the transient eddy forcing?*** Can we prove that we can represent the aggregate transient eddy effect in a linear model?



↳ To test this hypothesis, we take $\Delta(v'.\nabla q')$, scale it appropriately, and add it to the linear model as an extra transient forcing, giving:

$$\frac{\partial q}{\partial t} + \mathbf{v}.\nabla q = \mathcal{H} + f' - \Delta(\overline{v'.\nabla q'})$$

⇨ The resulting pattern resembles closely the difference between the runs in **SET1** and yet it is not the same kind of experiment at all. In **SET1**, it was the ***difference between two fully nonlinear turbulent experiments***, while **SET3** is ***a linear model response*** in which the turbulence has been added as a constant forcing.

We have thus proved that in a linear framework we can reproduce the effect of nonlinear transient eddies in the context of the response to a heating perturbation.

5.4.c) The importance of nonlinearity

⇨ And we arrive at fundamental considerations about the ***importance of non-linearity in low-frequency variability***.

• There is absolutely no question that ***the dynamics of the Atmosphere and Ocean are fundamentally nonlinear***. For example, we mentioned that the difference between cyclones and anticyclones is associated with non-linear dynamics (see **#GFD2.1a**).

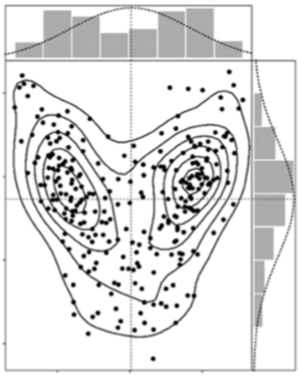
↳ Does this mean that the contribution of transient fluxes to low-frequency variability is automatically a nonlinear phenomenon? Or can it be thought of as a linear phenomenon? Changing something, the transients change one way, then changing it in the opposite way and the transients change in the opposite way? (*That would be linear*)

⇨ We are asking two different questions related to different time-scales:

▪ The first question is "are these eddies nonlinear?" and the answer is "yes, definitely!"

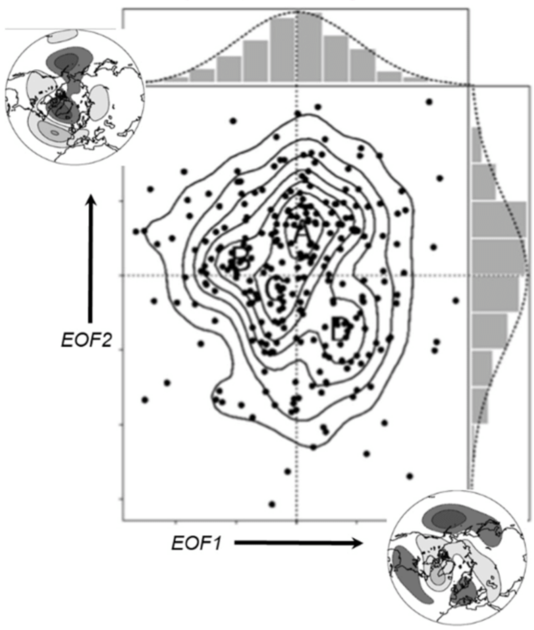
▪ The second question is "is the aggregate systematic effect of these eddies in modifying the atmospheric response to other types of forcing nonlinear?" and the answer is "yes, maybe".

↳ This is not the same question and there is no universal accord in the research community. There is a spectrum of opinions.



Supporters of nonlinear systems identify the Lorenz attractor system, the famous butterfly attractor, as a good model for the Atmospheric variability on low frequencies.

The figure to the left shows the Lorenz system mapped out in phase space. The many points show the instantaneous state of the system throughout a long integration of the simple Lorenz equations. They cluster very clearly onto two nodes with a bimodal distribution in one of the variables. Here we can identify two "regimes". The state goes from one regime to another and the time spent between the regimes remains quite small compared to the time spent in either one regime or the other.



This might be a useful way to think about the Atmosphere. On the right is an example of the (monthly mean) atmospheric variability represented in terms of the occurrence of two important patterns of low-frequency variability Pacific North America (PNA, horizontal axis) vs. the North Atlantic Oscillation (NAO, vertical axis). For each mode of variability, the associated PDF is also shown.

The question is "are the points clustering in specific regimes?" This is something that not everybody agrees about.

▪ It is possible that they are clustering in two regimes and we can think of transitions between regimes.

▪ Or it is possible that this impression of clusters is due to the sample of data that is finite (limited). In a finite sample of statistically random variables, you are always going to find some sort of clustering. So, it may also be appropriate to explain all this in a linear framework.

In linear dynamics, you will generally have Gaussian statistics and not the bimodal statistics associated with the Lorenz attractor.

Below is a linear equation that can be used, in which x is a state vector that represents the entire state of the atmosphere. Its development $\frac{dx}{dt}$ is determined by a linear operator, some external forcing, and some Gaussian noise:

$$\frac{dx}{dt} = Lx + f + B\eta$$

linear operators; state vector; external forcing; Gaussian noise

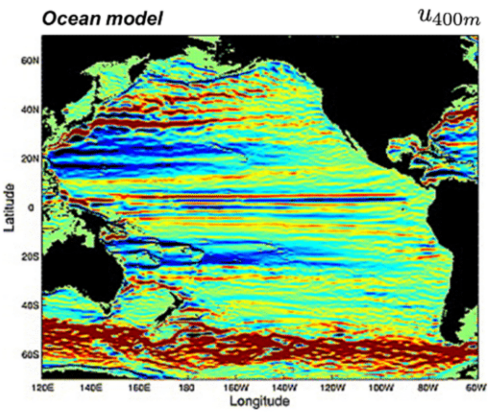
The Gaussian noise can be modified by another linear operator while remaining a linear system. If that second linear operator is independent of the flow x, then we still have Gaussian statistics everywhere. It is also possible to have non-Gaussian statistics with this linear system - skewed PDFs - provided that B is a function of the flow x. So, we can go a long way with such linear model to analyze the low-frequency variability.

GFD5.5: Zonal Jets and Turbulence

5.5.a) Zonal jets revisited: Ocean currents

⇨ The characteristics of the ocean circulation depend on the time scales considered:

- An instantaneous snapshot resembles a sea of eddies, i.e. little round blobs everywhere.
- A very long-time average primarily captures the anticyclonic gyres, Gulfstream, Kuroshio.
- The large-scale ocean circulation on a time-scale of a few months is characterized by zonal jets of alternating sign, eastward and westward jets separated by a typical length scale in the meridional direction.



This is illustrated in the picture on the left (from Richards et al., 2006) showing the zonal flow at 400 meters depth from a long simulation with a numerical model of the Ocean.

The situation is a little bit noisier in the observations, but the surface geostrophic flow and geostrophic vorticity, estimated from altimetric observations, also reveals these zonal jets.

5.5.b) Wave-Turbulence crossover

In this section, we look at the theory of turbulence at zonal jet length scales.

⇨ The Rossby radius is the length scale on which ***relative vorticity and vortex stretching*** make equal contributions to **potential vorticity** (see **#GFD1.2a**, and **#GFD3.4c**):

$$\nabla^2\psi \sim \frac{f^2}{gH}\psi \quad \Rightarrow \quad L \sim \frac{\sqrt{gH}}{f}$$

↳ The Rossby radius is the gravity wave speed divided by the Coriolis parameter.

⇨ Let's now consider larger length scales. We use the ***vorticity equation*** (see **#GFD1.3b**):

$$\frac{\partial\xi}{\partial t} + \mathbf{v}.\nabla\xi + \beta v = 0 \quad \rightarrow \quad \mathbf{v}.\nabla\xi \sim \beta v$$

⇨ The development of relative vorticity is balanced by the advection of relative vorticity and the advection of planetary vorticity. If the last two terms are of similar magnitude, a scale analysis yields to a length-scale on which it is true:

$$U\frac{U}{L^2} \sim \beta U \quad \rightarrow \quad L \sim \sqrt{\frac{u}{\beta}}$$

↳ L is the length scale on which advection of planetary and relative vorticity compare. Note that this very similar to the ***equatorial radius*** (see **#GFD4.3a**), except that in the square root we now have the actual flow speed and not the gravity wave speed. This length is called the Rhines scale, where Rossby waves give way to turbulence.

⇨ Let's now focus on what happens in the ***transition between Rossby waves and closed geostrophic eddies*** (turbulence). We compare the frequencies associated with these two processes: i.e., the frequency associated with (barotropic) Rossby waves (see **#GFD3.1c**) and a typical turbulence inverse timescale:

$$\omega = \frac{\beta l}{k^2} \sim u^* k$$

$k = (l, m)$ is the horizontal wavenumber and l is the zonal wavenumber ($k^2 = l^2 + m^2$)

↳ The frequency associated with turbulence is the length scale of that turbulence (the equivalent wavenumber) multiplied by a typical turbulent flow velocity scale.

⇨ Equating these two frequencies gives the spatial scales at which the two processes are of the same order:

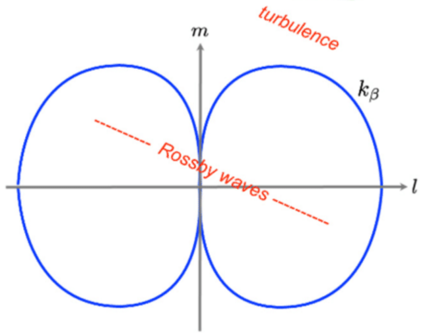
$$k^2 = \frac{\beta}{u^*} \cos\theta$$

$k = (l, m)$ forms an angle θ with the horizontal wavenumber l

↳ This equation is plotted in wavenumber space on the figure on the right. It looks like a dumbbell. The blue curve is the boundary between where the turbulence takes over and where Rossby waves dominate. It is ***anisotropic*** (≡ not isotropic).

• For larger scales, inside the blue dumbbell, there are Rossby waves (propagating westwards and mainly zonally).

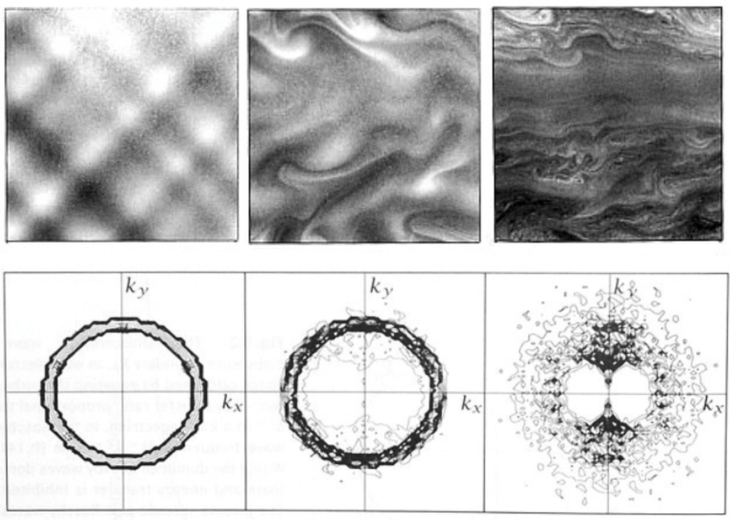
• For scales outside the dumbbell contour, geostrophic turbulence prevails.



⇨ Of particular interest are the points (positive and negative) where there the zonal wavenumber is small (large zonal scales) and there is a typical meridional length scale. Does this particular meridional length scale emerge from an analysis of the variability?. Yes, it does and it is the Rhines scale (see **#GFD5.5c**).

5.5.c) Collapse to zonal jets

⇨ Here are the results of an idealized numerical experiment in which variability naturally collapses into zonal jets.



• A turbulent model is initialized with only one single length-scale. The initial condition resembles a sort of grid lattice where the length equals the size of the grid. In (k_x, k_y) space, it is a circle (k=constant).

• Then, the flow gradually develops into turbulence and there will be scale interactions because the dynamics is nonlinear. The variability spreads across scales and the original circle in the wavenumber representation starts to spread out to other length scales.

• As the flow continues to develop, the variability spreads into a shape where turbulence is everywhere except inside the dumbbell associated with the Rossby wave regime.

↳ Most of the energy congregates to long zonal scales and a particular meridional length scale. This scale is the distance between zonal Jets that naturally emerges.

⇨ This is a neat theoretical account of the zonal jets observed in the Ocean.